Corporal Punishment, Child Development and Household Labor Supply*

Nikita Pavlov
Pennsylvania State University

October 2026

JEL Classification J13, D1

Keywords: Corporal punishment, Child development, Parenting styles

Abstract

This paper studies how corporal punishment affects child development and household resource allocation. I develop and estimate a dynamic model in which parents and children interact strategically. Child's human capital is assumed to feature two dimensions: cognitive capital and mental stability. The model separates the disciplinary role of punishment from its developmental costs. Counterfactual simulations of a ban on the corporal-punishment scheme imply large gains in children's mental stability, especially for children who would otherwise face positive punishment risk. Effects on cognitive capital differ between households. In particular, cognitive capital increases when parents are able to substitute toward non-violent supervision, but falls when the ban removes a disciplinary instrument without generating a compensating increase in productive inputs. However, replacing corporal punishment with more parental involvement reduces maternal labor supply and household income.

*

1 Introduction

Recent empirical studies[1] report consistent adverse associations between corporal punishment and child development. Guided by these empirical results, the federal government allocates significant funds each year to social programs promoting non-violent parenting. However, assessment of the effects of a switch to non-violent parenting on the basis of existing empirical studies is problematic. First, there is a causality problem. Spanking is correlated with a variety of often-omitted household state and choice variables. As a result, the effects of household decisions and characteristics merely correlated with spanking become attributed to spanking itself. Second, the consequences of a switch to non-violent parenting for the child's capital are complex and hard to capture in reduced form. A switch to non-violent parenting forces parents to adjust their parenting style to employ alternative means of control. This potentially changes the child's incentive scheme and parental time inputs, which could both have their own effects on the child's development. Third, child's capital is multidimensional. However, existing reduced-form studies are often limited to only one dimension and fail to consider spillover effects between children's non-cognitive and cognitive skills. Finally, reduced-form studies do not consider economic well-being of household members. A switch to non-violent parenting has a direct impact on parental time budget allocation. For example, more authoritative parenting styles often require parents to spend more time with the child. This could lead to changes in labor market participation and household income, directly affecting economic well-being of all household members.

To address the aforementioned issues, in this paper I build and estimate a dynamic model of child human capital formation which allows for corporal punishment and accounts for: 1) heterogeneity of preferences of both parents and the children across households, 2) strategic nature of interactions between parents and children, 3) multidimensionality of child's human capital, and 4) differences in labor market opportunities and income within and across households. In this model, parents choose how much productive time to spend with their child, how much leisure to enjoy and whether and how much to work. In turn, the child acts as a Stackelberg follower and allocates the rest of her time budget to self-investment, regular leisure and non-compliant leisure (misbehavior). Parents suffer utility losses from the child's misbehavior and can counter it either by tightening child's time budget by spending more time together or by implementing a corporal punishment scheme. Given the scheme, the realization of corporal punishment is stochastic. Realized punishment directly hurts child's utility, but as a Stackelberg follower child can alter expected punishment by misbehaving less. Thus, if implemented, the scheme directly discourages child from misbehaving and potentially allows parents to spend less time with the child for the same behavioral outcome. However, both implementation of the scheme and realized punishment have an impact on the child's cognitive and mental dimensions of human capital, which enter utilities of both parents and the child as public goods.

As a result, parental choice of whether to implement corporal punishment scheme involves a non-trivial trade-off, optimal solution in the face of which depends on preferences, wage offers and child's age. On the one hand, implementation of the corporal punishment scheme directly discourages misbehavior, indirectly incentivizes child's self-investment and allows parents to spend more time on leisure and work. On the other hand, it has direct adverse effects on the child's cognitive capital and mental health, with corresponding negative spillovers. A novel feature of my model is that it distinguishes between the introduction of the

[1] See Heilmann et al. (2021) for a recent meta-analysis; I discuss this literature in greater detail in Section 2.

corporal-punishment scheme itself and realized corporal punishment. The scheme is viewed as an ex-ante incentive device. In particular, even if punishment is not realized, its presence changes the child's expected payoff from misbehavior and hence the relative attractiveness of self-investment. However, the threat of physical punishment itself could also be harmful for child development. In turn, realized punishment is modeled as an ex-post outcome generated by the child's behavior under the scheme. Separating these two margins allows the model to distinguish three objects: the incentive value of the scheme, the developmental cost of announcing the scheme, and the developmental cost of realized physical punishment. To my knowledge, this separation and identification of corresponding effects is novel not only in the literature on structural modeling of child's human capital evolutions, but also in the literature on child development in general.

Similarly to other recent works on child human capital formation that utilize a structural approach, I use Method of Simulated Moments and U.S. Panel Study of Income Dynamics (PSID) data from 1986 to 2007 along with corresponding waves of Child Development Supplement (CDS) to estimate my model. Using CDS, I utilize detailed time diaries to gather information on time investments of all members of the household. To measure child's cognitive skills, I use Letter-Word test scores. To measure child's mental health level, I construct a Mental Stability test score on the basis of parental answers to CDS questions pertaining to the child's mental health. To obtain data on misbehavior, I use parental responses to CDS questions concerning child's behavioral problems. To obtain data on the usage of the corporal punishment scheme, I use the CDS question which puts parents in a hypothetical scenario where they should decide whether they would've spanked their child in response to her misbehavior. To obtain data on actually realized physical punishment, I use CDS question asking parents how many times their child was spanked last week which is then cross-verified by child's time diary reports.

Given the estimated model, I produce counterfactual analysis by comparing the outcomes of a direct ban on parental usage of the corporal punishment scheme to the baseline scenario. I find that a direct ban on the corporal punishment scheme leads to an average of 0.12 standard deviation increase in the final mental stability capital of children that were subjected to it, with an average of 0.2 standard deviation increase in the final mental stability capital for children that also experienced positive realized punishment. In terms of labor market effects, I find that a ban on the physical punishment scheme leads to an approximately 3% decrease in mothers' labor participation rate across households who practiced physical punishment with almost no change in father's participation rate. Moreover, mothers who stayed in the labor force decreased their median work hours by approximately 10.3%. Overall, this led to a 3% decrease in median income of the corresponding households. At the same time, the impact of the scheme on cognitive capital is ambiguous: while the direct impact of corporal punishment ban is positive, for some households it also led to decreased child's self-investment and decreased household income. As a result, the net change in cognitive capital generally depends on the relative magnitude of the aforementioned positive and negative effects, which differ between households.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the model primitives, including household preferences, the corporal-punishment scheme, and the laws of motion for cognitive capital and mental stability. Section 4 characterizes the household problem and describes the trade-off between corporal punishment and parental time as two alternative ways of discouraging misbehavior.

Section 5 describes the PSID/CDS data, the construction of cognitive and mental-stability measures, the measurement of misbehavior and corporal punishment, and the household choice variables used in estimation. Section 6 discusses identification and the simulated method of moments estimation procedure. Section 7 presents the model estimates. Section 8 evaluates the within-sample fit of the estimated model. Section 9 uses the estimated model to study a counterfactual ban on the corporal-punishment scheme and its effects on child development, parental labor supply, household income, and parental welfare. After that, Section 10 concludes.

2 Related Literature

My paper stands at the intersection of two strands of literature

The first strand views child development as an outcome of household resource allocation problem. Del Boca, Flinn, and Wiswall (2014) builds and estimates a dynamic model in which parental investments of time and money serve as inputs for child's human capital production function. Del Boca, Flinn, Verriest, and Wiswall (2025) extend this framework by allowing children themselves to be active decision makers. Verriest (2025) then allows for intra-household bargaining between spouses. More broadly, this line of work builds on the literature on dynamic skill formation, including Cunha and Heckman (2007) and Cunha, Heckman, and Schennach (2010).

The second strand studies parenting styles as equilibrium choices. Doepke and Zilibotti (2017, 2019) model parenting style as the result of strategic interaction between altruistic and paternalistic parents and children. Agostinelli, Doepke, Sorrenti, and Zilibotti (2020) extend the economic analysis of parenting by emphasizing the role of neighborhood and peer environments in child skill accumulation.

My paper is closely related to both. As in Del Boca, Flinn, and Wiswall (2014), Del Boca, Flinn, Verriest, and Wiswall (2025), and Verriest (2025), I model child development as an outcome of a dynamic household resource allocation problem. As in Doepke and Zilibotti (2017, 2019), I treat parenting style choice as an outcome of strategic interaction between parents and the child. This combination allows me to capture indirect equilibrium effects of a corporal punishment scheme ban on child's human capital and parental labor supply. In particular, in my model removal of the scheme as a parental disciplining option changes the child's incentives to misbehave and self-invest, leads to the adjustments in parental time allocation, and may crowd out parental labor supply and monetary investments. Consequently, these responses alter child's human capital production inputs, thus affecting child's human capital trajectory in both of its dimensions.

The third strand of literature worth mentioning is a large body of empirical works on corporal punishment and child outcomes. In particular, a broad set of studies documents adverse associations between physical punishment and children's mental health, cognitive, and educational outcomes. Gershoff (2002), Gershoff and Grogan-Kaylor (2016), Heilmann et al. (2021), and related reviews conclude that physical punishment is systematically associated with worse child outcomes. Other longitudinal studies, such as MacKenzie et al. (2012, 2013), study spanking in early childhood and find adverse associations with later behavioral and cognitive outcomes. This evidence is important because it documents robust patterns in the data and motivates the aforementioned policy interest in non-violent parenting.

However, most of this evidence is difficult to interpret as causal. Parents do not randomly choose whether to use corporal punishment. Spanking is correlated with the child's underlying behavior, parental preferences, and other household characteristics. Moreover, the relationship is dynamic: child misbehavior may induce

punishment, punishment may change future child behavior, and parents may simultaneously adjust time investments, labor supply, and other disciplinary tools. Even longitudinal specifications that control for lagged outcomes cannot fully separate the direct effect of punishment from selection into punishment and from the equilibrium household responses that follow when punishment is used or removed.

3 Model

3.1 Primitives and Environment

Each household consists of two parents and a single child who make decisions throughout M periods, $t \in \{1, ..., M\}$, with t_M being thought of as a final period of child development. In each period, parents have one joint utility function which depends positively on the mother's and father's leisure (respectively, l_1 and l_2), private consumption (c), the child's cognitive capital (k_h) and the child's mental stability capital (k_p). It depends negatively on the child's non-compliant leisure (l_{cd}). Parents spend their respective time budgets (T_i) on leisure, productive time with child (τ_i) and labor (h_i). There is no lending or borrowing. Household income consists of labor ($w_1 \cdot h_1 + w_2 \cdot h_2$) and non-labor components. It is spent on private consumption and monetary investments into child's capital (e_c). The child acts as a Stackelberg follower. Parental time with child enters child's time budget (T_c), the rest of which is allocated by child freely between self-investment (τ_c), regular leisure (l_c) and non-compliant leisure. In each period, parents can implement a corporal punishment scheme (1_s), which enables a Poisson distribution of executed punishment (s) with mean proportional to the amount of non-compliant leisure. Expected executed punishment enters child's utility and disincentivizes her from choosing non-compliant leisure. The production technology of both cognitive and mental stability capital has Cobb-Douglas structure. Both announced scheme and executed punishment enter the evolution process of mental stability[2]. Scheme announcement also directly enters the evolution process of cognitive skills [3], with executed punishment affecting it only indirectly through the spillover effect from mental stability. Apart from that, cognitive skill evolution also depends on parental time investments, parental monetary investments, child's self-investment, and non-compliant leisure.

Given the primitives, the overall timing of the model in period t could be described as follows:

1. Given preferences, wage offers, and non-labor income, parents decide whether to implement corporal punishment scheme ($1_{s,t}$) and how to allocate time budgets (T_1, T_2) between respective amounts of leisure ($l_{1,t}, l_{2,t}$), work ($h_{1,t}, h_{2,t}$) and productive time with child ($\tau_{1,t}, \tau_{2,t}$);

2. Given parental choices of time allocation and of whether to use the scheme, child allocates the rest of her time budget ($T_{c,t} - \tau_{1,t} - \tau_{2,t}$) between leisure ($l_{c,t}$), non-compliant leisure ($l_{cd,t}$) and self-investment ($\tau_{c,t}$);

3. Given the scheme ($1_{s,t}$) and non-compliant leisure ($l_{cd,t}$) choices, physical punishment is realized;

[2]This specification is motivated by evidence linking physical punishment to worse child mental-health and behavioral outcomes (e.g., Gershoff, 2002; Gershoff and Grogan-Kaylor, 2016; Heilmann et al., 2021), as well as by the broader childhood-adversity literature emphasizing that both threatening environments and realized harmful events may affect children's mental health (e.g., McLaughlin et al., 2014; Sheridan and McLaughlin, 2014).

[3]The scheme can be interpreted as a proxy for a broader harsh or violent parenting style. Its effect on cognitive skills allows to potentially account for meta-analytic evidence that maltreated youth perform significantly worse in terms of academic-performance measures (McGuire and Jackson, 2018).

4. Next period cognitive ($k_{h,t+1}$) and mental stability ($k_{p,t+1}$) capitals are formed.

In the next subsections I describe the components of the model in more detail.

3.2 Instantaneous Utility: Child and Parents

In each period $t \in \{1, ..., M\}$, each child has Cobb-Douglas instantaneous utility depending on regular leisure ($l_{c,t}$), non-compliant leisure ($l_{cd,t}$), cognitive capital ($k_{h,t}$) and mental stability capital ($k_{p,t}$). In an absence of corporal punishment:

$$u_{c,t} = \lambda_1 \log l_{c,t} + \lambda_2 \log(1 + l_{cd,t}) + \lambda_3 \log k_{h,t} + \lambda_4 \log k_{p,t} \tag{1}$$

Parents have joint utility function which depends on leisure of both parents ($l_{1,t}$ and $l_{2,t}$, respectively), private consumption (c_t), both dimensions of child's human capital, and the child's non-compliant leisure. That is:

$$u_{p,t} = \alpha_1 \log l_{1,t} + \alpha_2 \log l_{2,t} + \alpha_3 \log c_t + \alpha_4 \log k_{h,t} + \alpha_5 \log k_{p,t} - \alpha_6 \log(1 + l_{cd_t}) \tag{2}$$

I allow for heterogeneity in preferences across households but not across time. Sum of preference weights is without loss of generality assumed to be one for both child and parents ($\sum_{i=1}^{6} \alpha_i = 1, \sum_{k=1}^{4} \lambda_k = 1$). Notice that non-compliant leisure ($l_{cd,t}$) is a good for child but is an anti-good for parents. Assumed functional form allows for child's corner solution in terms of non-compliant leisure. As will be evident in the following sections, this simplifies solution and facilitates identification.

3.3 Corporal Punishment

In each period $t \in \{1, ..., M\}$ parents may choose to implement a corporal punishment scheme ($1_{s,t}$) to discourage their child from choosing non-compliant leisure. Implementation of the scheme has fixed utility cost v_t for parents which is allowed to vary with time and with parental education (subscript dropped for notational simplicity). With that in mind, total instantaneous utility of parents in period t becomes:

$$U_{p,t} = u_{p,t} - 1_{s,t} v_t \tag{3}$$

where $u_{p,t}$ is given by (2). If parents choose to implement the scheme, actual realized punishment s_t is assumed to be a random draw from a Poisson distribution dependent on the amount of non-compliant leisure chosen. That is,

$$s_t \sim_d \text{Pois}\big(b \cdot \log(1 + l_{cd,t})\big). \tag{4}$$

where b is a distribution parameter that is the same for all households. As a rational Stackelberg follower, in the presence of the scheme, the child makes choices taking into account the expected amount of realized punishment she will get. That is, if parents are using the scheme, the child's instantaneous utility becomes

$$U_{c,t} = u_{c,t} \; - \; 1_{s,t}\, \delta\, \mathbb{E}[s_t] = u_{c,t} \; - \; 1_{s,t}\, \delta \cdot b \log\big(1 + l_{cd,t}\big), \tag{5}$$

where δ is some fixed disutility from one unit of punishment and $u_{c,t}$ is given by (1).

Notice that the structure of the model distinguishes between announced scheme and executed punishment. This important detail allows us to account for a variety of scenarios observed in the data, such as: 1) parents who implement the scheme may not use physical punishment because kids are well-behaved ($1_{s,t} = 1$, $l_{cd,t} = 0$, $s_t = 0$); 2) parents who implement the scheme aren't always using physical punishment even on consistently misbehaving kids ($1_{s,t} = 1$, $l_{cd,t} > 0$, $s_t = 0$). However, those few instances in the data when well-behaved kids were subjected to physical punishment could not be rationalized by my model and were instead attributed to measurement error: although it is straightforward to allow for such scenarios in the model, executed punishment in the absence of a disciplining motive is child abuse which is not the focus of this work.

3.4 Technology of Child's Human Capital Formation

Child has two dimensions of human capital: mental stability and cognitive skills. Technology parameters are allowed to vary with time but are the same across households. Formation of cognitive skills is given by the following law of motion:

$$\begin{aligned}\log\big(k_{h,t+1}\big) \;&= R_{h,t}(1_{s,t}) \;+\; \gamma_{1,t}\log(\tau_{1,t}) \;+\; \gamma_{2,t}\log(\tau_{2,t}) \;+\; \gamma_{3,t}\log(\tau_{c,t}) \\ &\quad -\; \gamma_{4,t}\log\big(1 + l_{cd,t}\big) \;+\; \gamma_{5,t}\log\big(e_{c,t}\big) \;+\; \gamma_{6,t}\log\big(k_{h,t}\big) \;+\; \gamma_{7,t}\log\big(k_{p,t}\big).\end{aligned} \tag{6}$$

In the above, $R_{h,t}$ is a total factor productivity which is also allowed to depend on whether the corporal punishment scheme was implemented. Cognitive skills formation also depends on parental time investments ($\tau_{1,t}$ and $\tau_{2,t}$), child's self-investment ($\tau_{c,t}$), non-compliant leisure ($l_{cd,t}$), monetary investments in child's quality ($e_{c,t}$), its own current level ($k_{h,t}$), and mental stability ($k_{p,t}$). I allow for some technology parameters (namely, $\gamma_{1,t}$ and $\gamma_{2,t}$) to depend on parental educational attainment, but for notational simplicity corresponding subscript was dropped.

Formation of mental stability is given by the following law of motion:

$$\log\big(k_{p,t+1}\big) \;= R_{p,t}(1_{s,t}) - \delta_{1,t}s_t \;+\; \delta_{2,t}\log\big(k_{p,t}\big). \tag{7}$$

In the above, $R_{p,t}$ is a total factor productivity which is allowed to depend on whether the corporal punishment scheme was implemented ($1_{s,t}$). Additionally, mental stability evolution depends on the amount of actually executed punishment (s_t) and on its own current level ($k_{p,t}$).

In addition to the usage of the corporal punishment scheme and mental stability spillover, I allow cognitive skill formation to depend directly on non-compliant leisure. This term is included to avoid confounding the effect of the corporal-punishment scheme with the effect of child misbehavior. Since parents are more likely to use the scheme when the child is more non-compliant, omitting $l_{cd,t}$ would load part of the cognitive effect of misbehavior onto the scheme-dependent TFP term. Including $l_{cd,t}$ therefore helps separate the developmental effect of the disciplinary environment from the effect of the behavior that triggers discipline.

The sign of technology parameters is assumed positive for all inputs except for non-compliant leisure ($-\gamma_{4,t}$) and executed punishment ($-\delta_{1,t}$). I did not, however, impose the direction of effect of scheme ($1_{s,t}$) on respective TFP parameters.

3.5 Time and Budget Constraints

For each period $t \in \{1, 2, ..., M\}$, each parent is assumed to have time endowment T_p which is spent freely on leisure ($l_{i,t}$), work ($h_{i,t}$) and productive time with child ($\tau_{i,t}$):

$$T_p = l_{i,t} + h_{i,t} + \tau_{i,t},\ i \in \{1, 2\} \tag{8}$$

In turn, each child is allocated $T_{c,t}$. Unlike parental time endowment which is assumed to be stable across all periods, $T_{c,t}$ is allowed to vary with the average amount of formal schooling observed for that age in the data. $T_{c,t}$ is spent on time with parents ($\tau_{1,t}$ and $\tau_{2,t}$) which the child cannot alter, self-investment ($\tau_{c,t}$), non-compliant leisure ($l_{cd,t}$), and regular leisure ($l_{c,t}$):

$$T_{c,t} = \tau_{1,t} + \tau_{2,t} + \tau_{c,t} + l_{cd,t} + l_{c,t} \tag{9}$$

Notice that by putting pressure on child's budget by increasing their time spent with child, parents can suppress non-compliant leisure even without the usage of corporal punishment scheme. This creates an important trade-off discussed in more detail in Section 4.2.

Finally, given the labor choices $h_{1,t}$ and $h_{2,t}$, total household income consists of labor income ($h_{1,t}w_{1,t} + h_{2,t}w_{2,t}$) and non-labor income ($I_t$). No borrowing or lending is allowed. Parents allocate total household income between private consumption c_t and investments in child quality $e_{c,t}$:

$$h_{1,t}w_{1,t} + h_{2,t}w_{2,t} + I_t = c_t + e_{c,t} \tag{10}$$

In the above, prices of private consumption and monetary investments were assumed to be equal.

3.6 Terminal Period Utility

It is assumed that $t = M$ is the last period of child development, after which both parents and the child enjoy the resulting accumulated capital in perpetuity. In accordance with instantaneous utilities specified in (1) and (2), terminal period utility for child and parents becomes, respectively:

$$V_{c,M+1} = \frac{\lambda_3 \log k_{h,t} + \lambda_4 \log k_{p,t}}{1 - \beta_{c,M+1}}, \tag{11}$$

$$V_{p,M+1} = \frac{\alpha_4 \log k_{h,t} + \alpha_5 \log k_{p,t}}{1 - \beta_p}. \tag{12}$$

Although child's discount factor is exogenous, I allow it to vary with age and parental education by using DFVW (2025) estimates. This, together with the heterogeneity of preferences across households, creates unique terminal values for each family.

4 Household Problem and Parenting Style

4.1 Dynamic Problem

Given the primitives of the model described in the previous section, in each period $t \in \{1, \ldots, M\}$ household plays a Stackelberg game with parents being the first movers and the child being the follower. In

particular, taking the parental choice vector $a_{p,t} = \{\tau_{1,t}, \tau_{2,t}, h_{1,t}, h_{2,t}, l_{1,t}, l_{2,t}, c_t, e_t, 1_{s,t}\}$ and household state variables $\Gamma_t = \{k_{h,t}, k_{p,t}, w_{1,t}, w_{2,t}, I_t\}$ as given, the child allocates the rest of her time endowment $T_{c,t} - \tau_{1,t} - \tau_{2,t}$ to self-investment, non-compliant leisure and regular leisure: $a_{c,t} = \{\tau_{c,t}, l_{cd,t}, l_{c,t}\}$. That is, child solves:

$$\begin{aligned} V_{c,t}(\Gamma_t|a_{p,t}) = \max_{a_{c,t}|a_{p,t}} & U_{c,t}(a_{c,t}|a_{p,t}) \; + \; \beta_{c,t} E_t\big(V_{c,t+1}(\Gamma_{t+1}|a_{p,t+1})|\Gamma_t\big) \\ \text{s.t.} \quad & T_{c,t} - \tau_{1,t} - \tau_{2,t} = \tau_{c,t} + l_{cd,t} + l_{c,t} \\ & k_{h,t+1}, k_{p,t+1} \text{ given by (6), (7)} \end{aligned} \tag{13}$$

where $U_{c,t}$ is given by (5). Notice that, for the sake of generality, the child's problem in (13) is written as depending on the full parental choice vector a_p. However, in reality, solution to (13) is affected only by $\tau_{1,t}$, $\tau_{2,t}$, and $1_{c,t}$. Next, as a Stackelberg leader, parents utilize the knowledge of their child's reaction function and solve their optimization problem by choosing $a_{p,t}|a^*_{c,t}(a_{p,t})$:

$$\begin{aligned} V_{p,t}(\Gamma_t) = \max_{a_{p,t}|a^*_{c,t}(a_{p,t})} & U_{p,t}(a_{p,t}) \; + \; \beta_p E_t\big(V_{p,t+1}(\Gamma_{t+1})|\Gamma_t\big) \\ \text{s.t.} \quad & T_p = \tau_{i,t} + h_{i,t} + l_{i,t}, \; i \in 1, 2 \\ & h_{1,t} w_{1,t} + h_{2,t} w_{2,t} + I_t = c_t + e_t \\ & k_{h,t+1}, k_{p,t+1} \text{ given by (6), (7)} \end{aligned} \tag{14}$$

where $U_{p,t}$ is given by (3).

Given (13) and (14), for each $t \in \{1, ..., M\}$ events in the model proceed as follows:

1. Wages $\{w_{1,t}, w_{2,t}\}$ and non-labor income $\{I_t\}$ are realized;
2. Parents strategically choose $a^*_{p,t}$ by maximizing (14);
3. Given $a^*_{p,t}$, child chooses $a^*_{c,t}$ by maximizing (13);
4. Punishment s_t is realized according to (4);
5. Cognitive capital $k_{h,t+1}$ and mental stability capital $k_{p,t+1}$ are formed on the basis of (6) and (7), respectively, and period ends.

With the convenient functional-form assumptions imposed on the utility functions and the punishment distribution, as in DFVW (2025), it is possible to obtain conditional closed-form solutions for all household choice variables as functions of parental time investments $\tau_{1,t}$ and $\tau_{2,t}$. However, unlike DFVW (2025), household choices do not depend on current and future levels of capital. Therefore, the model is solved by forward induction similarly to DFW (2014).

4.2 Corporal Punishment Tradeoff

As was mentioned earlier, parents have two possible mechanisms to discourage their child's choice of non-compliant leisure. First, they can tighten the child's time budget by increasing time spent together by

raising $\tau_{1,t}$ and $\tau_{2,t}$. Second, they can implement a corporal punishment scheme $1_{s,t}$. The latter changes the child's incentives while potentially allowing parents to allocate more time on work or leisure for the same behavioral outcome. However, it comes at the cost of directly hurting the child's human capital. To better illustrate how this key tradeoff drives parenting style choice, here I compare the child's choice vector in the presence and in the absence of an implemented corporal punishment scheme.

For any $t \in \{1, ..., M\}$, fix $\tau_{1,t}, \tau_{2,t}$ and first assume that no corporal scheme is used, $1_{s,t} = 0$. By plugging capital evolution laws (6) and (7) into value function given in (13) and iterating forward, child's problem is equivalent to

$$\begin{aligned} \max_{\tau_{c,t}, l_{cd,t}, l_{c,t} | \tau_{1,t}, \tau_{2,t}, 1_{s,t}} \quad & \lambda_1 \log l_{c,t} + (\lambda_2 - \lambda_3 \tilde{\gamma}_{4,t}) \log(1 + l_{cd,t}) + \lambda_3 \tilde{\gamma}_{3,t} \log \tau_{c,t} \\ \text{s.t} \quad & T_{c,t} - \tau_{1,t} - \tau_{2,t} = l_{c,t} + l_{cd,t} + \tau_{c,t} \end{aligned} \tag{15}$$

In the above expression, $-\lambda_3 \tilde{\gamma}_{4,t}$ and $\lambda_3 \tilde{\gamma}_{3,t}$ capture the additional utility generated by, respectively, one extra unit of non-compliant leisure and one extra unit of the child's self-investment, through their effects on the future path of capital, with all future values discounted to the present.

Given the corner solution option for non-compliant leisure $l_{cd,t}$, problem (15) could lead to two cases depending on model parameters and parental choices of time investments. These cases are captured by systems (16) and (17) below. As can be seen, there are two ways in which, in the absence of a corporal punishment scheme, a child could choose a corner solution in terms of misbehavior. First, it is possible that the child does not enjoy non-compliant leisure enough, $\lambda_2 \leq \lambda_3 \tilde{\gamma}_{4,t}$. Given that preferences are heterogeneous across households, some children indeed may choose not to engage in non-compliant leisure even in the absence of parental involvement. Second, parents may tighten child's budget so that she does not have enough time for non-compliant leisure, $\tilde{T}_{c,t} \leq \frac{\lambda_1 + \lambda_3 \tilde{\gamma}_{3,t}}{\lambda_2 - \lambda_3 \tilde{\gamma}_{4,t}}$. The amount of joint time needed for the latter generally depends on child's preferences. However, this additional time with child usually comes at a cost of foregone leisure, work hours, and child's crowded-out self-investment[4].

1. For $\lambda_2 > \lambda_3 \tilde{\gamma}_{4,t}$ and $\tilde{T}_{c,t} > \frac{\lambda_1 + \lambda_3 \tilde{\gamma}_{3,t}}{\lambda_2 - \lambda_3 \tilde{\gamma}_{4,t}}$,

$$\begin{cases} l_{c,t} = \frac{\lambda_1}{\Delta_{0,t}} (1 + \tilde{T}_{c,t}) \\ \tau_{c,t} = \frac{\lambda_3 \tilde{\gamma}_{4,t}}{\Delta_{0,t}} (1 + \tilde{T}_{c,t}) \\ l_{cd,t} = \frac{\lambda_2 - \lambda_3 \tilde{\gamma}_{3,t}}{\Delta_{0,t}} (1 + \tilde{T}_{c,t}) \end{cases} \tag{16}$$

2. For $\lambda_2 \leq \lambda_3 \tilde{\gamma}_{4,t}$ or $\tilde{T}_{c,t} \leq \frac{\lambda_1 + \lambda_3 \tilde{\gamma}_{3,t}}{\lambda_2 - \lambda_3 \tilde{\gamma}_{4,t}}$,

$$\begin{cases} l_{c,t} = \frac{\lambda_1}{\lambda_1 + \lambda_3 \tilde{\gamma}_{3,t}} \tilde{T}_{c,t} \\ \tau_{c,t} = \frac{\lambda_3 \tilde{\gamma}_{3,t}}{\lambda_1 + \lambda_3 \tilde{\gamma}_{3,t}} \tilde{T}_{c,t} \\ l_{cd,t} = 0 \end{cases} \tag{17}$$

[4] In particular, it comes at a cost when joint parental time $\tau_{1,t} + \tau_{2,t}$ is higher than what parents would have chosen in the absence of a disciplining motive. In other words, when $\tau_{1,t} + \tau_{2,t}$ is higher than the sum of $\tau^*_{1,t}$ and $\tau^*_{2,t}$, which solve problem (15) with $l_{cd,t}$ set to 0.

where $\Delta_{0,t} = \lambda_1 + \lambda_2 - \lambda_3\tilde{\gamma}_{3,t} + \lambda_3\tilde{\gamma}_{4,t}$ is the usual Cobb-Douglas denominator and $\tilde{T}_{c,t} = T_{c,t} - \tau_{1,t} - \tau_{2,t}$ is a free time budget child has left after spending time with parents.

Now, consider child's problem in the presence of a corporal punishment scheme, $1_{s,t} = 1$. This new problem differs from (15) since now child is additionally discouraged from using non-compliant leisure in two ways. First, there is direct disutility from expected punishment $-\delta \cdot b \log(1 + l_{cd,t})$ as specified in (5). Second, there is indirect disutility $-\lambda_4\tilde{\delta}_{2,t} \cdot b \log(1 + l_{cd,t})$ from expected damage of punishment to future mental stability k_p and cognitive capital k_h obtained from iterating forward (7), (6) and (13). As a result, problem (15) becomes:

$$\begin{aligned} \max_{\tau_{c,t}, l_{cd,t}, l_{c,t} | \tau_{1,t}, \tau_{2,t}, 1_{s,t}} \quad & \lambda_1 \log l_{c,t} + (\lambda_2 - \lambda_3\tilde{\gamma}_{4,t} - b(\delta + \lambda_4\tilde{\delta}_{2,t})) \log(1 + l_{cd,t}) + \lambda_3\tilde{\gamma}_{3,t} \log \tau_{c,t} \\ \text{s.t} \quad & T_{c,t} - \tau_{1,t} - \tau_{2,t} = l_{c,t} + l_{cd,t} + \tau_{c,t} \end{aligned} \tag{18}$$

As before, it leads to two cases:

1. For $\lambda_2 > \lambda_3\tilde{\gamma}_{4,t} + b(\delta + \lambda_4\tilde{\delta}_{2,t})$ and $\tilde{T}_{c,t} > \frac{\lambda_1 + \lambda_3\tilde{\gamma}_{3,t}}{\lambda_2 - \lambda_3\tilde{\gamma}_{4,t} - b(\delta + \lambda_4\tilde{\delta}_{2,t})}$,

$$\begin{cases} l_{c,t} = \frac{\lambda_1}{\Delta_{1,t}}(1 + \tilde{T}_{c,t}) \\ \tau_{c,t} = \frac{\lambda_3\tilde{\gamma}_{4,t}}{\Delta_{1,t}}(1 + \tilde{T}_{c,t}) \\ l_{cd,t} = \frac{\lambda_2 - \lambda_3\tilde{\gamma}_{3,t} - b(\delta + \lambda_4\tilde{\delta}_{2,t})}{\Delta_{1,t}}(1 + \tilde{T}_{c,t}) \end{cases} \tag{19}$$

2. For $\lambda_2 \leq \lambda_3\tilde{\gamma}_{4,t} + b(\delta + \lambda_4\tilde{\delta}_{2,t})$ or $\tilde{T}_{c,t} \leq \frac{\lambda_1 + \lambda_3\tilde{\gamma}_{3,t}}{\lambda_2 - \lambda_3\tilde{\gamma}_{4,t} - b(\delta + \lambda_4\tilde{\delta}_{2,t})}$,

$$\begin{cases} l_{c,t} = \frac{\lambda_1}{\lambda_1 + \lambda_3\tilde{\gamma}_{3,t}}\tilde{T}_{c,t} \\ \tau_{c,t} = \frac{\lambda_3\tilde{\gamma}_{3,t}}{\lambda_1 + \lambda_3\tilde{\gamma}_{3,t}}\tilde{T}_{c,t} \\ l_{cd,t} = 0 \end{cases} \tag{20}$$

where $\Delta_{1,t} = \lambda_1 + \lambda_2 - \lambda_3\tilde{\gamma}_{3,t} - b(\delta + \lambda_4\tilde{\delta}_{2,t}) + \lambda_3\tilde{\gamma}_{4,t}$.

Notice that now both preference and free time thresholds for non-corner choice of non-compliant leisure became larger. Moreover, for any fixed $\tau_{1,t}$ and $\tau_{2,t}$, we have $l_{cd,t}(\tau_{1,t}, \tau_{1,t}, 1_{s,t} = 1) \leq l_{cd,t}(\tau_{1,t}, \tau_{1,t}, 1_{s,t} = 0)$ and $\tau_{c,t}(\tau_{1,t}, \tau_{1,t}, 1_{s,t} = 1) \geq \tau_{c,t}(\tau_{1,t}, \tau_{1,t}, 1_{s,t} = 0)$, with equalities only when the corner solution is chosen. That is, the corporal punishment scheme potentially allows parents to force their child into better behavioral outcomes for less time spent, while simultaneously stimulating her self-investment. However, even in the absence of non-compliant leisure, and hence zero punishment according to (4), corporal punishment scheme implementation is still never free in terms of capital accumulation because of its adverse effects on the respective TFPs.

Thus, corporal punishment scheme choice presents households with a trade-off: its implementation potentially allows households to force the child to simultaneously have lower levels of non-compliant leisure and higher levels of self-investment, while allowing parents to spend more time on work and leisure. However, this happens at a cost of direct TFP damage across both dimensions of child's capital evolution with possibility (in case of $l_{cd,t} > 0$) of additional damage to both dimensions of capital through realized punishment.

5 Data and Descriptive Statistics

To estimate the model, I use data from Panel Study of Income Dynamics (PSID) from 1986 to 2007 accompanied by three corresponding waves (1997, 2002, 2007) of Child Development Supplement (CDS). The main PSID dataset presents a longitudinal study of U.S. households and features detailed information on economic outcomes (wages, labor supply, non-labor income) and socio-economic characteristics (age, fertility, education) of the members of each household tracked. Three waves of CDS supplement PSID by providing detailed information on parenting choices (e.g., corporal punishment preferences and outcomes), child's time allocation in the form of time diaries, and coherent measures of child's cognitive skills, behavioral outcomes and mental well-being.

In order to be included in my final sample, a PSID household had to satisfy certain criteria, namely: include two parents, one of whom had to be registered as a primary caregiver; have at most two children with at least one participating in two consecutive CDS waves[5]; have non-missing data on model outcomes, which include choices of parents and child, as well as measures of cognitive and non-cognitive skills. Overall, this left me with 290 households out of the initial sample of 1591. These 290 household records include 690 observations of CDS reports across three waves. Below I discuss particular aspects of data used in more detail.

5.1 Cognitive Skills: Test Scores and Latent Ability

Similarly to previous works exploring child human capital formation on PSID data (e.g., Del-Boca et al. (2014, 2025), Verriest (2025)), I use Letter-Word (LW) test scores to anchor the child's latent cognitive capital. Letter-Word test was administered on every wave of CDS for each child starting from the age of 3, and featured 57 questions of increasing difficulty assessing symbolic learning and reading identification skills. Figure 1 below depicts evolution of LW Score with child's age. As can be seen, there is a clear positive age trend indicating accumulation of human capital as children get older. Table 1 presents a breakdown of mean and variance of LW Score by three age bins. Observe that not only does mean LW score increase with age but also its variance falls, indicating a ceiling effect of the maximum score.

[5]When two children are present in the household, similarly to Del-Boca et al. (2025), I pick the youngest child since usually the data on the youngest child is available through more CDS waves

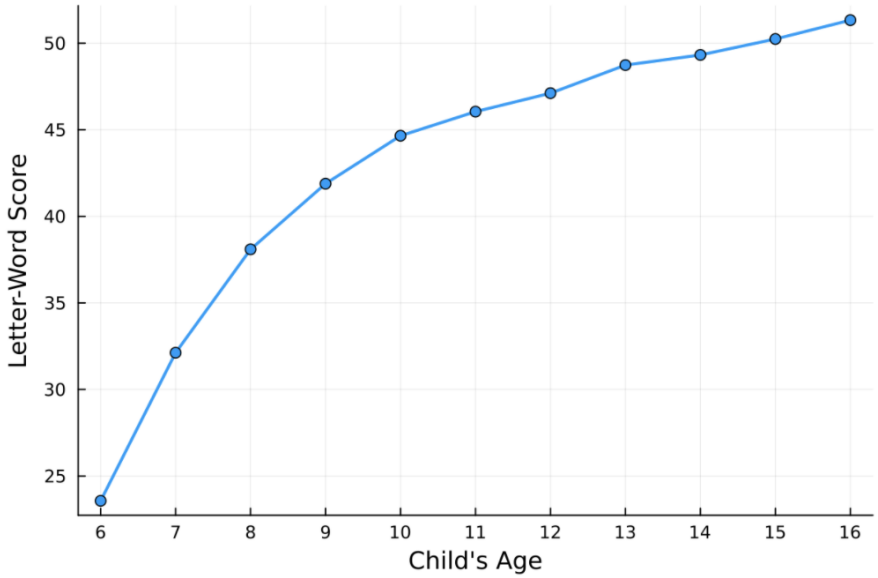


Figure 1: Evolution of LW Scores with Child's Age

	$t \in [6, 8]$	$t \in [9, 12]$	$t \in [13, 16]$
$\mathbb{E}[LW_t]$	32.01	45.00	49.76
σ_{LW_t}	8.93	5.56	4.16
N	163	253	212

Table 1: Sample mean and standard deviation of LW Score by Child's Age

Note that while on the side of data cognitive skills are measured by test scores, on the side of model they are measured by cognitive capital k_h. Hence, to fit the model we need to define mapping between observed test scores and cognitive capital. To do that, similarly to Verriest (2025), I utilize the psychometric approach of Item Response Theory (IRT). In particular, for each of 57 binary items of LW test (1 for correct answer, 0 for incorrect one) IRT defines probability of correct answer (Item Characteristic Curve, ICC) for child i as a logistic function of her latent ability θ_i as follows:

$$p_j(\theta_i) = \frac{\exp(a_j(\theta_i - b_j))}{1 + \exp(a_j(\theta_i - b_j))} \tag{21}$$

where a_j and b_j are, respectively, discriminating power and difficulty parameters of item j. Discriminating power controls how sharply probability of correct answer increases with the ability, whereas difficulty tells where along the ability scale an item provides most information (that is, where $p_j = 0.5$). As a result, this specification utilizes information on the difficulty and responsiveness to ability of each item instead of treating every question identically. Overall, given the sample of 690 Letter-Word records across three waves of CDS, the resulting model has 114 ($\{a_j, b_j\}_{j=1}^{57}$) + 690 ($\{\theta_i\}_{i=1}^{690}$) unknown parameters which are estimated via the usual procedure of iterated Maximum Likelihood. Results are pictured in Figures 2 and 3 below.

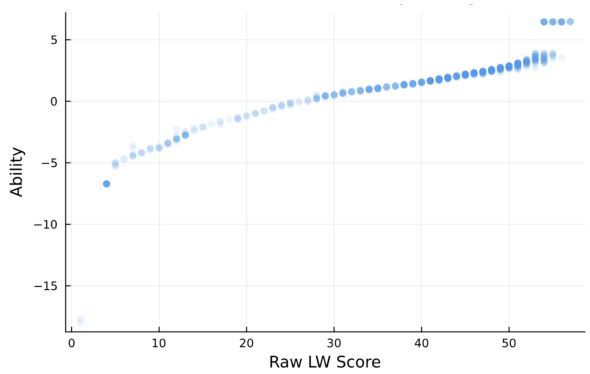


Figure 2: Scatterplot of Raw LW Score and Latent Ability

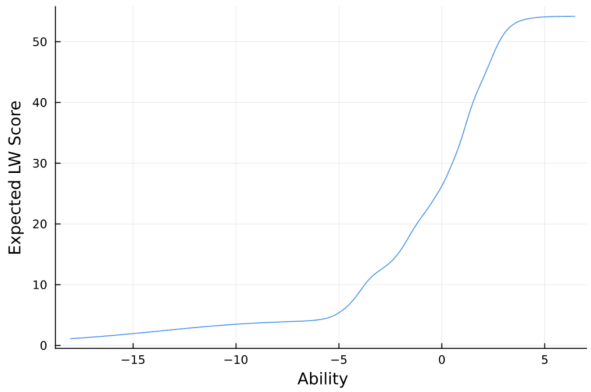


Figure 3: Latent Ability and Projected LW Score

Note that resulting estimates of latent abilities have $(-\infty, +\infty)$ domain with arbitrary zero midpoint and unit of measurement which could not be directly estimated. This makes the ordering of latent abilities unique up to a linear transformation, and their map to cognitive capital requires normalization. The latter is usually referred to as "anchoring" problem in the literature (e.g., Cunha et al. (2021)). As a result, in order to transform latent ability θ_i to cognitive capital $k_{h,i}$ I, similarly to Verriest (2025), assume the following map:

$$k_{h,i} = \exp(\theta_i) \tag{22}$$

which has an advantage of being strictly positive and hence making the law of cognitive capital evolution (6) well-defined for all observations.

5.2 Mental Stability: Test Scores and Latent Ability

In order to measure mental stability capital I utilize questions on child's mental well-being as reported by parents under Behavior Problem section of CDS. Behavior Problem section was administered under each CDS wave for every child from the age of six. It is dedicated to construction of Behavior Problem Index (BPI) which is an aggregate measure of external and internal behavioral outcomes consisting of 30 questions. Out of these 30, I pick 18 binary questions that are directly related to mental well-being and construct Mental Stability Score (MS Score). Questions picked, evolution of mean mental stability with age, and some descriptive statistics could be found in Table 2 below. Figure 2 depicts evolution of MS Score with age indicating high persistence of average mental stability level as children grow. Table 3 captures mean and standard deviation of MS Score across three age bins. As can be seen, with age mental stability becomes more dispersed.

Table 2: Questions used to construct Mental Stability Test

#	Question Text
1	(He/She) has sudden changes in mood or feeling
2	(He/She) feels or complains that no one loves him/her
3	(He/She) is rather high strung, tense and nervous
4	(He/She) is too fearful or anxious
5	(He/She) has difficulty concentrating, cannot pay attention for long
6	(He/She) is easily confused, seems to be in a fog
7	(He/She) has trouble getting along with other children
8	(He/She) feels worthless or inferior
9	(He/She) is not liked by other children
10	(He/She) has difficulty getting his/her mind off certain thoughts (has obsessions)
11	(He/She) is overly dependent on others
12	(He/She) is withdrawn, does not get involved with others
13	(He/She) cries too much
14	(He/She) is too shy or timid
15	(He/She) is too fearful of new situations or people
16	(He/She) worries too much
17	(He/She) has sudden unexplained fears
18	(He/She) is nervous or tense

	$t \in [6, 8]$	$t \in [9, 12]$	$t \in [13, 16]$
$\mathbb{E}[MS_t]$	14.39	13.74	13.93
σ_{MS_t}	3.07	3.48	3.64
N	163	253	212

Table 3: Sample mean and standard deviation of MS Score by Child's Age

To map MS Score to the latent mental stability capital $k_{p,i}$ I utilize the exact same procedure as for Letter-Word Score described in section 5.1. Results are reported in Figures 4 and 5 below.

To address anchoring problem, I again assume that there is a simple exponential map from latent mental stability γ_i to mental stability capital $k_{p,i}$:

$$k_{p,i} = \exp(\gamma_i) \tag{23}$$

5.3 Corporal Punishment Scheme 1_s and Executed Punishment s

In order to gain data on whether the corporal punishment scheme was used by parents, and if there was any executed punishment realized, I utilize CDS questions on parental disciplining preferences and outcomes, as well as detailed child's time diaries.

Baseline binary question on usage of corporal punishment scheme $1_{s,t}$ is featured on the parental questionnaire of all three CDS waves for children from age 3 and is formulated as follows:

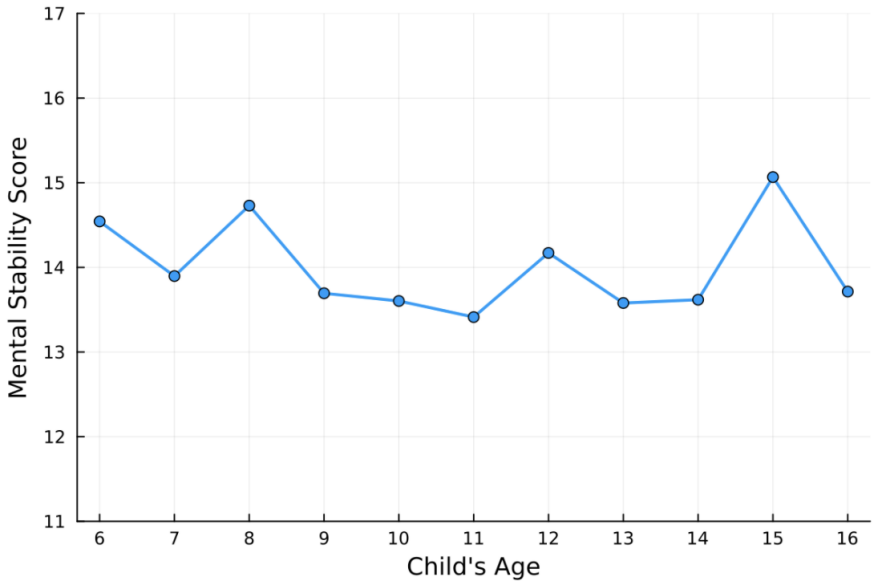


Figure 4: Evolution of MS Scores with Child's Age

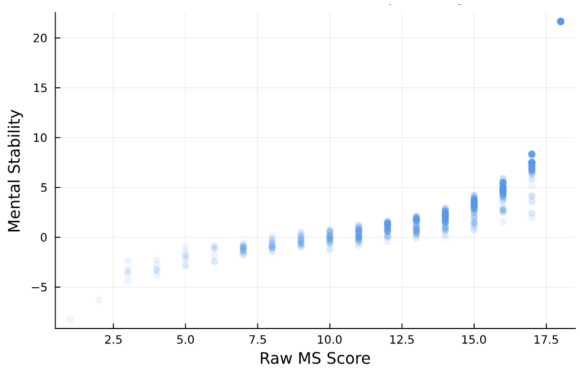


Figure 5: Scatterplot of Raw MS Score and Latent Mental Stability

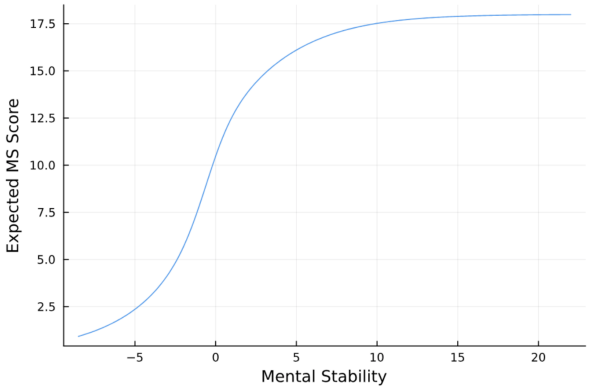


Figure 6: Latent Mental Stability and Projected MS Score

(I) *Most children get so angry at their parents that they say things like "I hate you," swear in a temper tantrum, or hit you. If CHILD did any of these, would you spank (him/her)?*

Given the sensitivity of the topic, it is very likely that resulting data from the above question would feature a lot of noise because of potential misreports. In order to mitigate this issue, I cross-verify it by another question on preference for corporal punishment, which is formulated as follows:

(II) *If children are seriously misbehaving it is best to spank them?*

with answers ranging from "strongly agree" (5) to "strongly disagree" (1). This question is also featured across all three CDS waves but its availability is more limited since it was asked only for children of age from 6 to 10.

In addition to that measure, to verify parental preferences on corporal punishment, I use a revealed-

preference approach by checking if the child was actually spanked in the year of the corresponding CDS wave. To do so, I use the CDS question on the child's age at first and last spanking as reported by parents. This recurring question was included in all CDS waves, and if the age of first or last spanking as reported at some wave corresponded to the child's age at the time of the CDS interview in period t, I impute $1_{s,t} = 1$ regardless of answers to questions (I) and (II) in that period. The same was done if the executed punishment s_t, construction of which is discussed below, was positive. Resulting descriptive statistics with breakdown by child's age and parental education could be found in Table 4. As expected, we observe a decrease in violent parenting with both age and educational attainment.

	$t \in [6, 8]$	$t \in [9, 12]$	$t \in [13, 16]$
$P(I_{s,t} > 0)$	0.37	0.25	0.12
$P(I_{s,t} > 0 \mid ed_2 \leq 12)$	0.43	0.30	0.16
$P(I_{s,t} > 0 \mid ed_2 > 12)$	0.32	0.21	0.08
N	167	253	212

Table 4: Usage of Corporal Punishment by Child's Age and Father's High School Graduation. ed_2 denotes educational attainment in years.

To gain data on executed punishment s for each household, I use CDS question on the number of times child was spanked last week as reported by parents, which also was collected across all CDS waves for every child from the age of 3:

(III) *How many times in the past week have you spanked CHILD?*

To cross-verify answers to question (III), I analyze child's time diaries provided in every wave of CDS. For two random days within a week before interview (one weekday and one weekend), it presents a thorough report of all activities the child undertook during the day with no gaps. These activities were then divided into categories by PSID personnel. Resulting time diaries feature a time category of being physically disciplined. Using this information, s was constructed as follows: if the number of times child was spanked as reported by parents in (III) was smaller than the sum of times being physically disciplined as reported by child in time diaries, I increased this child's s by that difference. Resulting descriptive statistics could be found in Table 5.

	$t \in [6, 8]$	$t \in [9, 12]$	$t \in [13, 16]$
$E(s_t \mid 1_{s,t} = 1)$	0.53	0.40	0.24
$\sigma_{s_t \mid 1_{s,t}=1}$	0.80	0.89	0.52
$P(s_t > 0 \mid 1_{s,t} = 1)$	0.37	0.27	0.20

Table 5: Mean, Standard Deviation, and Percent of families with positive executed punishment by Child's Age. By construction, only families with $1_{s,t} = 1$ could have $s_t > 0$.

5.4 Non-compliant Leisure l_{cd}

Since I don't have information on what is considered non-compliant leisure in each particular household, I cannot construct l_{cd} on the basis of the child's time diaries. Instead, similarly to cognitive skills and mental well-being, for each child I construct Behavior Problem (BP) score from parental reports on the child's compliance, extract a latent misbehavior parameter, and assume that there is some unknown map from latent misbehavior to time spent on non-compliant leisure.

To construct BP score, I use 9 questions on external behavioral problems under the Behavior Problem section of CDS. Each question asks parents about the frequency of certain type of misbehavior exhibited by child with answers ranging from "often" (3) to "never" (1). As a result, potential BP score ranges from 9 to 27. Full list of questions utilized could be found in Table 6 below.

#	Question Text
1	(He/She) cheats or tells lies
2	(He/She) argues too much
3	(He/She) bullies or is cruel or mean to others
4	(He/She) is disobedient
5	(He/She) does not seem to feel sorry after (he/she) misbehaves
6	(He/She) is impulsive, acts without thinking
7	(He/She) is stubborn, sullen, or irritable
8	(He/She) has a strong temper, loses temper easily
9	(He/She) is destructive with toys or other objects

Table 6: Questions used to construct Behavior Problem Test

Given 9 BP questions for each child, I then use factor analysis to deduce latent misbehavior ξ (common factor) for each child. Results are presented in Figure 7, while Table 7 gives descriptive statistics of ξ with breakdown by age and parental usage of corporal scheme 1_s. Table 7 reveals some interesting patterns: first, children that are subjected to corporal scheme exhibit higher levels of misbehavior; second, there is an increase in average misbehavior of children that are not subjected to corporal scheme with age. From Table 4 it is clear that the latter is likely a consequence of decrease of usage of corporal scheme by parents with child's age.

	$t \in [6,8]$	$t \in [9,12]$	$t \in [13,16]$
$E(\xi_t)$	0.00	-0.08	0.01
$E(\xi_t \mid 1_{s,t} = 1)$	0.31	0.16	0.20
$E(\xi_t \mid 1_{s,t} = 0)$	-0.18	-0.16	-0.01
σ_{ξ_t}	0.95	0.86	0.88
N	167	253	212

Table 7: Mean and Standard Deviation of Latent Misbehavior by Child's Age.

Finally, to obtain non-compliant leisure from the estimates of latent misbehavior, I assume that there is an

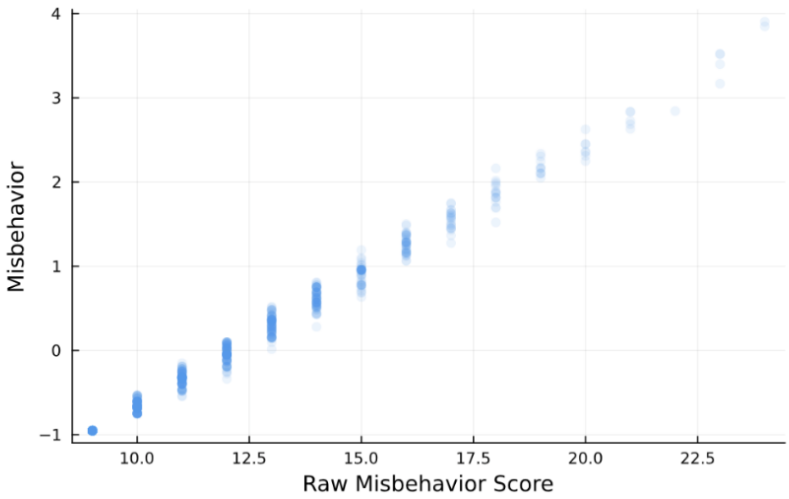


Figure 7: Scatterplot of Latent Misbehavior ξ and Raw BP Score. Raw BP Score ranges from 9 to 27

unknown map from ξ to l_{cd} of the following form

$$l_{cd}(\xi) = \max(A + B \cdot \xi, 0). \tag{24}$$

Such functional relationship imposes a cut-off rule for l_{cd} by assuming corner solution for every child in the data with latent misbehavior below a certain threshold. Particular value of the threshold is imposed at the level of median latent misbehavior, roughly corresponding to 1 out of 18 points in BP score:

$$A = -B \cdot \text{median}(\xi_t).$$

Although it would've been more intuitive to impose normalization at the level of 0 BP score (no misbehavior reported by parents), this makes the sample size of children with the corner solution relatively small. In turn, by setting the threshold at the median level of latent misbehavior we're greatly improving the stability of estimation while only marginally affecting the interpretation of the latent misbehavior measure.
Given the normalization, slope parameter B is then estimated along with the other model parameters.

Alternatively, I could've potentially avoided the last step by changing the model structure and assuming that the child directly chooses the amount of non-compliance ξ instead of time spent on non-compliance l_{cd}. However, framing of non-compliance choice in terms of time budget allocation has its benefits: first, it implicitly allows for substitution effects between non-compliance and self-investment, which helps the model to better fit patterns observed in the data (see section 5.6); second, it presents parents with natural mechanism of discouraging non-compliance by increasing time spent with child, which becomes a non-violent alternative to corporal punishment scheme and leads to one of the central trade-offs of my model (see section 4.2 for discussion).

5.5 Time Investments

Methodology of construction of time investment variables is very similar to Del-Boca et al. (2025). First, I analyze both weekday and weekend CDS time diaries for children in each wave and separate all activities of

the child into four mutually exclusive categories: active mother's involvement, active father's involvement, active involvement of both parents, and parental absence during activity. To obtain total time an activity takes during the week, I multiply its recorded duration by 5 for weekdays and by two for weekends, and sum it together. If only the mother was actively involved, corresponding duration is counted towards active time with mother $\tau_{1,t}$; if only the father was actively involved, corresponding duration is counted towards active time with father $\tau_{2,t}$; if both parents were actively involved, then the total corresponding duration is divided evenly between $\tau_{1,t}$ and $\tau_{2,t}$. Finally, if none of the parents were involved, and the activity falls under one of the productive investment categories (e.g., studying, reading, solving puzzles, etc.), then it counts towards the child's self-investment $\tau_{c,t}$. The full list of activities that fall under productive investment categories can be found in Appendix B. Thus, total $\tau_{1,t}$, $\tau_{2,t}$ and $\tau_{c,t}$ comprise the sum of the duration of all time diary activities transformed and allocated by the above procedure. Table 8 below presents descriptive statistics of the resulting variables. As can be seen, time investment patterns display familiar trends documented in the

	$t \in [6, 8]$	$t \in [9, 12]$	$t \in [13, 16]$
$E(\tau_{1,t})$	17.62	16.86	13.35
$\sigma_{\tau_{1,t}}$	8.88	10.44	11.51
$E(\tau_{2,t})$	7.73	7.07	5.55
$\sigma_{\tau_{2,t}}$	6.15	6.82	6.27
$E(\tau_{c,t})$	1.55	3.82	5.18
$\sigma_{\tau_{c,t}}$	2.58	4.87	6.38
N	167	253	212

Table 8: Mean and Standard Deviation of Family Members' Time Investments.

literature: first, active time with parents decreases with age, while self-investment increases; second, there is considerable variability in time choices across households even conditional on child's age.

5.6 Labor Supply, Wages and Non-Labor Income

Methodology of construction of labor supply, wages and non-labor income is identical to Del-Boca et al. (2025). To construct these variables for each household, I use the main PSID dataset from 1997 to 2007, which is collected biennially. For weekly labor supply of each parent, I sum total yearly hours of all jobs including overtime and divide by 52. To obtain hourly wages, I divide total labor income by total labor hours. Finally, for weekly non-labor income I subtract labor income from total income and divide by 52. Descriptive statistics for labor supply and wages can be found in Tables 9 and 10, respectively.

Statistic	Data	Statistic	Data
$E[h_{2,t} \mid h_{2,t} > 0,\ t \in [6,8]]$	43.638	$P(h_{2,t} > 0 \mid t \in [6,8])$	0.973
$E[h_{2,t} \mid h_{2,t} > 0,\ t \in [9,12]]$	44.519	$P(h_{2,t} > 0 \mid t \in [9,12])$	0.987
$E[h_{2,t} \mid h_{2,t} > 0,\ t \in [13,16]]$	43.780	$P(h_{2,t} > 0 \mid t \in [13,16])$	0.990
$E[h_{1,t} \mid h_{1,t} > 0,\ t \in [6,8]]$	31.616	$P(h_{1,t} > 0 \mid t \in [6,8])$	0.797
$E[h_{1,t} \mid h_{1,t} > 0,\ t \in [9,12]]$	30.414	$P(h_{1,t} > 0 \mid t \in [9,12])$	0.811
$E[h_{1,t} \mid h_{1,t} > 0,\ t \in [13,16]]$	32.932	$P(h_{1,t} > 0 \mid t \in [13,16])$	0.846
$P(h_{1,t} > 0, h_{2,t} > 0 \mid t \in [6,8])$	0.770	$P(h_{1,t} > 0, h_{2,t} > 0 \mid t \in [9,12])$	0.800
$P(h_{1,t} > 0, h_{2,t} > 0 \mid t \in [13,16])$	0.836		

Table 9: Mother's (h_1) and father's (h_2) labor supply by child's age (t).

Statistic	Data	Statistic	Data
$E[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [18,32]]$	13.568	$E[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [33,37]]$	17.077
$E[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [38,42]]$	18.961	$E[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [43,65]]$	22.126
$\sigma[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [18,32]]$	5.433	$\sigma[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [33,37]]$	11.332
$\sigma[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [38,42]]$	11.475	$\sigma[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [43,65]]$	13.374
$E[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [18,32]]$	20.367	$E[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [33,37]]$	22.668
$E[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [38,42]]$	27.335	$E[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [43,65]]$	32.424
$\sigma[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [18,32]]$	11.327	$\sigma[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [33,37]]$	13.667
$\sigma[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [38,42]]$	16.392	$\sigma[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [43,65]]$	21.208
$E[w_{1,t} \mid w_{1,t} > 0,\ ed_1 = 12, t_1 < 40]$	14.622	$E[w_{1,t} \mid w_{1,t} > 0,\ 13 \le ed_1 \le 15, t_1 < 40]$	16.601
$E[w_{1,t} \mid w_{1,t} > 0,\ ed_1 \ge 16, t_1 < 40]$	21.440	$E[w_{1,t} \mid w_{1,t} > 0,\ ed_1 = 12, t_1 \ge 40]$	16.827
$E[w_{1,t} \mid w_{1,t} > 0,\ 13 \le ed_1 \le 15, t_1 \ge 40]$	19.948	$E[w_{1,t} \mid w_{1,t} > 0,\ ed_1 \ge 16, t_1 \ge 40]$	25.683
$E[w_{2,t} \mid w_{2,t} > 0,\ ed_2 = 12, t_2 < 40]$	19.533	$E[w_{2,t} \mid w_{2,t} > 0,\ 13 \le ed_2 \le 15, t_2 < 40]$	23.379
$E[w_{2,t} \mid w_{2,t} > 0,\ ed_2 \ge 16, t_2 < 40]$	33.764	$E[w_{2,t} \mid w_{2,t} > 0,\ ed_2 = 12, t_2 \ge 40]$	23.784
$E[w_{2,t} \mid w_{2,t} > 0,\ 13 \le ed_2 \le 15, t_2 \ge 40]$	29.140	$E[w_{2,t} \mid w_{2,t} > 0,\ ed_2 \ge 16, t_2 \ge 40]$	40.268
$\sigma[w_{1,t} \mid w_{1,t} > 0]$	12.141	$\sigma[w_{2,t} \mid w_{2,t} > 0]$	18.632
$corr(w_{1,t}, w_{2,t})$	0.308		

Table 10: Mother's (w_1) and father's (w_2) wages by age and educational attainment in years $\{t_i, ed_i\}$.

6 Identification and Estimation

This section summarizes the intuition behind the model identification and estimation. The full econometric specification is described in Appendix A, while a more detailed discussion of the identification argument and estimation procedure is provided in Appendix B. The goal of this section is therefore to explain how the observed variation in household choices, punishment, and capital outcomes disciplines the main blocks of model parameters.

The identification argument is organized around three blocks of parameters: preferences and corporal-punishment-scheme parameters, the cognitive and mental-stability technologies, and the income processes. The model is estimated by simulated method of moments. For any trial parameter vector, I simulate household histories over the child-development period, solve the household problem, and simulate the same moments that are observed in the data. The parameter estimates minimize the weighted distance between the observed and simulated moments.

6.1 Preferences and the Corporal-Punishment Scheme

Identification of the preference and punishment parameters relies primarily on household choices observed under three different scenarios. First, consider periods in which the child chooses zero non-compliant leisure. In this case, utility terms associated with non-compliant leisure and realized punishment do not affect the household's optimal choices. As a result, we obtain the same system of optimal choices as in DFVW (2025), which, given the income process and human capital production technologies, is invertible in preference

parameters up to a normalization. Similarly to DFVW (2025), moments of the child's self-investment choices then provide information on the relative importance of cognitive capital and regular leisure in the child's utility. In turn, parental labor supply, time investments, and child expenditures allow us to identify parental preferences for leisure, consumption, and child capital.

Now consider periods in which the child chooses a positive amount of non-compliant leisure, but parents do not implement the corporal punishment scheme. Unlike the previous case, household choices now also depend on the child's preference for non-compliant leisure and parental disutility from the child's misbehavior. However, since the scheme is not implemented, punishment does not affect the household problem. Thus, given the preference parameters identified from the first case, variation in the child's non-compliant leisure, parental time investments, labor supply, and child expenditures provides information on these two additional preference components.

Finally, consider periods in which the child chooses positive non-compliant leisure and parents implement the corporal punishment scheme. In this case, the amount of expected punishment depends on the child's misbehavior, which, in turn, is affected by parental time investments. Since realized punishment directly affects the child's mental stability, parental time now has an additional benefit: by reducing the child's misbehavior, parents can also reduce expected punishment and the corresponding damage to future mental stability. As a result, parental choices of time investments and labor supply in this case provide information on the value parents attach to the child's mental stability capital. On the child's side, the presence of the scheme introduces an additional disutility from non-compliant leisure through expected punishment. As can be seen from (4) and (5), the resulting deterrence effect depends on three unknown components: the relationship between misbehavior and expected punishment, the child's direct utility cost of punishment, and the child's valuation of mental stability. To identify them separately, I first use a moment of expected punishment to identify the parameter governing the punishment distribution. Given this parameter, I then use variation in misbehavior choices with the child's age under the scheme to distinguish between the remaining two components.

The above identification argument is exact only in the case of homogeneous preferences. In the estimated model, however, preferences are allowed to vary across households. As a result, households observed in each of the three cases are selected on their preferences, and conditional choice means alone are not sufficient for identification. To account for this, I also match the probabilities of observing each case, together with variances and correlations of household choices. In particular, the choice probabilities provide information on the distribution of preferences that leads households to choose each of the three behavioral outcomes, while conditional means capture the average choices within each group. Finally, variance and covariance moments provide additional information on the variance-covariance matrix of the preference distribution.

6.2 Technology Parameters

Given preferences, punishment parameters and the income process, I next identify the parameters governing the evolution of child's human capital. Recall that the model has two dimensions of child capital: cognitive skills and mental stability. As specified in (6), cognitive capital accumulation depends on parental time investments, child's self-investment, monetary investments, non-compliant leisure, previous levels of cognitive capital and mental stability, and whether the corporal punishment scheme was implemented. To

identify the corresponding technology parameters, I use normalized covariance moments between cognitive outcomes and these variables, both contemporaneously and five years forward. The former describe the relationship between household inputs and cognitive capital observed in the same period, while the latter relate current inputs and capital stocks to cognitive outcomes in the next observed CDS wave. Similarly, I use corresponding moments to identify the parameters governing the evolution of mental stability.

Apart from the input coefficients, both capital production functions contain total factor productivity (TFP) terms which determine the average level of capital not explained by the observed household inputs and state variables. Recall that in my model TFP is allowed to depend on whether parents implement the corporal punishment scheme. In particular, for each dimension of child capital, I distinguish between baseline TFP and an additional productivity term associated with the scheme. To identify baseline TFP parameters, I use mean cognitive and mental stability outcomes by child's age. In turn, to identify the scheme-dependent TFP parameters, I use corresponding conditional mean outcomes by age and corporal punishment scheme usage. Given the input coefficients, these moments provide information on the differences in capital production between households that implement the scheme and those that do not within the same age group.

6.3 Income Processes

The income processes consists of non-labor income and wage offers for both parents. The non-labor income process is estimated separately from the main structural model using a two-step procedure. First, I use a logit regression to estimate the probability of positive non-labor income. Then, conditional on non-labor income being positive, I use a linear regression to estimate its level.

Unlike non-labor income, the wage process is estimated jointly with the other model parameters. Since labor supply is an endogenous household decision, the sample of observed wages is subject to selection. To account for this, in each simulation I first generate potential wage offers for both parents, and then parental labor supply choices determine whether their wages are observed. As a result, I calculate simulated wage moments only for parents with positive labor supply, similarly to how the corresponding moments are constructed in the data. In particular, to identify the wage process parameters, I use mean wages and their standard deviations by parental age, mean wages by parental education, overall wage variance, and the within-household correlation between mothers' and fathers' wages.

6.4 Estimation and Sensitivity of Preference Covariances

I estimate the model using the Method of Simulated Moments (MSM). Given a trial parameter vector, I simulate household histories, including preferences, wage offers, non-labor income, initial states, household choices, punishment, and the evolution of cognitive capital and mental stability. I then calculate the corresponding simulated moments and compare them with those observed in the data. The parameter estimates are obtained by minimizing the weighted distance between the two sets of moments. For this purpose, I use a diagonal weighting matrix, with weights determined by the estimated sampling variances of the corresponding moments.

One concern with the estimation is that, given the heterogeneity of preferences allowed by the model, some parameters of the preference covariance matrix may be weakly identified by the chosen moments. To address this issue, after obtaining an initial set of estimates, I conduct a sensitivity analysis of the estimated covariance parameters. In particular, starting from the estimated parameter vector, I perturb each covariance

parameter and examine how these changes affect the simulated moments and the MSM objective. If changing a particular covariance parameter has essentially no effect on the objective function, I treat this parameter as weakly identified and impose a restriction on it in the next estimation step. I then re-estimate the model and repeat the procedure until all remaining unrestricted covariance parameters have a non-negligible effect on the targeted moments.

7 Model Estimates

Detailed model estimates are presented in Section C of the appendix. Here I discuss some of interesting aspects of the estimated parameters.

7.1 Preference Distribution

Since household preferences are generated by first drawing a latent normal vector and then applying a nonlinear preference transformation, the distribution parameters reported are simulation-based distribution of the transformed utility coefficients. Tables 26 and 27 summarize the simulated distribution of transformed preference coefficients.

Table 26 reports mean and variance of simulated utility weights. The estimates imply that parents place substantial weight on parental leisure and consumption. The mean transformed weight on the father's leisure is 0.322, followed by consumption, 0.269, and the mother's leisure, 0.262. Parents also place a sizable weight on the child's cognitive capital, with mean 0.221. The mean weight on the child's mental stability is smaller, 0.030, while the disutility weight on non-compliant leisure is 0.085.

The child's preferences display a different pattern. The largest mean weight is attached to regular leisure, 0.548, followed by non-compliant leisure, 0.375, and cognitive capital, 0.343. The mean weight on mental stability is also small, 0.028.

A notable feature of the estimates is that child preference weights are much more dispersed than parental preference weights. This larger variance indicates that the model requires substantial heterogeneity in how children respond to similar household environments to fit the moments. In particular, variation in child preferences helps the model match high variance of self-investment and non-compliant leisure that cannot be explained by parental choices or household resources alone.

Table 27 reports the correlation matrix of simulated utility weights distribution. Overall, we observe several economically meaningful patterns.

First, the child's preference for non-compliant leisure is negatively correlated with the child's weight on cognitive capital. This covariance pattern helps rationalize why children with high levels of self-investment tend to have low levels of misbehavior.

Second, the child's preference for non-compliant leisure is positively correlated with parental distaste for non-compliant leisure. This pattern helps explain why children in households that use the corporal punishment scheme may nevertheless display higher levels of non-compliant leisure than children in the no-scheme households. In particular, the scheme is more likely to be used in households facing children with a stronger underlying taste for misbehavior. These children are also less attached to future-oriented state variables, making them more tolerant of the developmental repercussions of punishment.

Third, the child's preference for non-compliant leisure is negatively correlated with both parental and

child preferences for cognitive and mental-stability capital. This, together with the two previous facts, helps separate households along a parenting style dimension. In one group, parents place high value on child's human capital, and misbehavior is disciplined primarily through higher time investments. In the other group, children have a stronger taste for non-compliant leisure, while parents get greater disutility from misbehavior and suffer less from corporal punishment scheme developmental cost. For these households, announcing the corporal punishment scheme may become a more relevant tool than time investments.

In turn, some correlations are less intuitive. For example, the weight on father's leisure is strongly positively correlated with the parental weight on child cognitive capital. This should not be interpreted as implying that father's leisure and child development are complements. Rather, it implies that the model fits a data pattern according to which fathers who work less tend to spend more time both on leisure and on productive time with the child.

Finally, other correlations are mechanical. As such, an almost perfect co-movement of parental preference for cognitive capital and mental stability capital comes from very small estimated variances of the respective latent parameters. As a result, the only source of variation in both transformed coefficients comes from the same denominator.

7.2 Corporal Punishment Scheme and Technology Estimates

Sections C.2 and C.3 of the Appendix present the model estimates of corporal punishment scheme and technology parameters.

The technology estimates show clear age patterns in the productivity of different investment inputs. In particular, the estimated productivity of child's self-investment increases from 0.049 in the youngest age interval to 0.052 in the middle interval and 0.069 in the oldest interval. In contrast, the productivities of parental time investments decrease over the development period. The implied return to mother's time falls from 0.028 to 0.025 and then to 0.022, while that of father's time falls from 0.015 to 0.013 and then to 0.011. Thus, the estimates imply a gradual shift in cognitive capital production away from parental time and toward child's own self-investment.

The estimates also distinguish between two channels through which the corporal punishment scheme affects child development: announcement of the scheme itself and realized corporal punishment. For mental stability, the estimated damage comes primarily from realized punishment, especially in the first two age intervals. However, in the oldest age interval, announcement of the scheme itself also has a sizable negative effect. For cognitive capital, I allow for a direct effect of scheme announcement, which is estimated to be largely negative across all three age intervals. In turn, realized punishment is allowed to affect cognitive capital only indirectly through the mental stability spillover. The estimated spillover is sizable at early ages but becomes close to zero later in the development period.

The estimates also imply that non-compliant leisure itself has almost no direct effect on cognitive capital. Thus, the model does not attribute lower cognitive outcomes among misbehaving children to a large direct productivity loss from non-compliant leisure itself. Instead, the developmental consequences of misbehavior operate mainly through the disciplinary response associated with it.

Finally, recall that the total cost of the corporal punishment scheme consists of direct utility costs and developmental costs. The direct utility cost of implementing the scheme does not change monotonically with

child's age. However, this non-monotonicity is partly offset by the opposite age pattern in developmental costs. In particular, in age intervals where the direct utility cost is lower, the developmental cost of the scheme is larger, and vice versa.

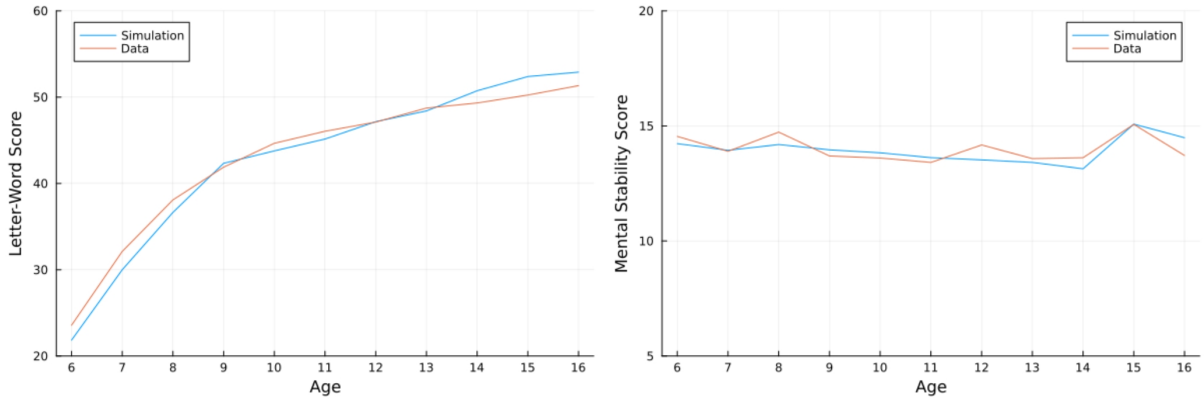
8 Within-Sample Fit

This section presents the within-sample fit of the estimated model. Appendix D reports the full set of targeted moments together with their simulated counterparts. Here I discuss selected moments pertaining to cognitive capital, mental stability, parental labor supply and wages, corporal punishment scheme usage, and time allocation.

8.1 Age Profiles

Before discussing the selected targeted moments, I compare model-generated age profiles with those observed in the data. Recall that many estimation moments are constructed for age intervals, rather than for each child's age separately. Thus, comparing the profiles at each age allows us to additionally examine whether the estimated model matches the developmental patterns in the data. These figures summarize the within-sample fit visually; they do not constitute an additional set of moment conditions.

Figure 8 presents observed and simulated profiles for both dimensions of child's human capital. As can be seen from Panel (a), the model closely matches the steep increase in Letter-Word scores over childhood and adolescence. Simulated scores are slightly lower than those in the data at the youngest ages and slightly higher at the oldest ages. However, the overall upward age profile is reproduced well. In turn, Panel (b) shows that the estimated model also reproduces the relatively flat profile of mental stability with child's age. Although mental stability in the data varies more from one age to another, the simulated levels remain close to the observed levels throughout the age range.

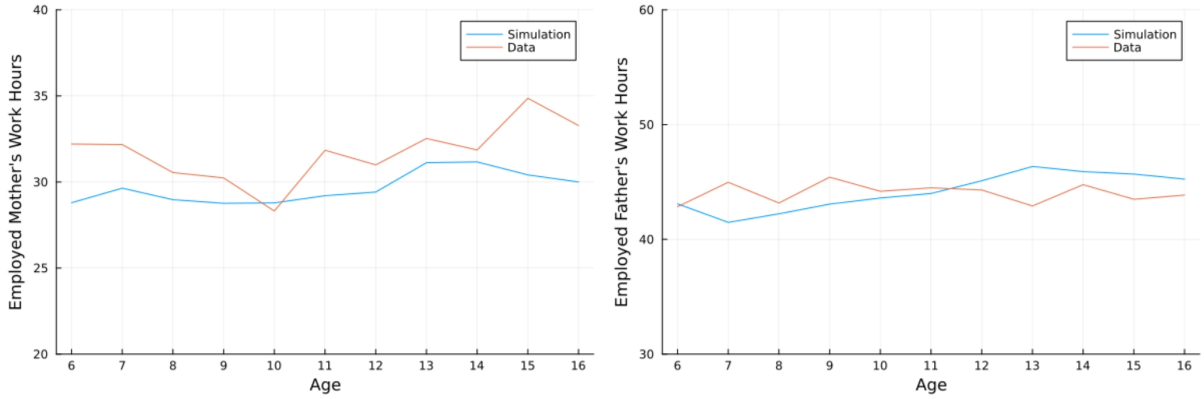


(a) Letter-Word score

(b) Mental stability score

Figure 8: Model Fit: Child Development Outcomes by Age

Figure 9 reports the age profiles of employed parents' work hours. The model captures the broad level of fathers' work hours and reproduces their relatively flat age profile. However, for mothers, the model slightly underpredicts the overall level of work hours and smoothens some of the age-specific fluctuations observed in the data. In particular, the data show larger movements in mothers' work hours at older child ages than the simulated series.



(a) Mother's work hours

(b) Father's work hours

Figure 9: Model Fit: Employed Parents' Work Hours by Child Age

Figure 10 reports the age profiles of parental and child time investments. The model captures the broad decline in parental time investment with the child's age, especially for mothers. Mother's time investment is high at young ages and declines as the child gets older in both the data and the simulation. Father's time investment is lower in level and also declines with age, although the model somewhat smoothens the sharp decline observed in the data at the oldest ages. The model also reproduces the increasing profile of child self-investment, which is one of the central developmental patterns in the data. The fit is less exact at some ages, but the simulation captures the main shift from parental time toward child self-investment as children age.

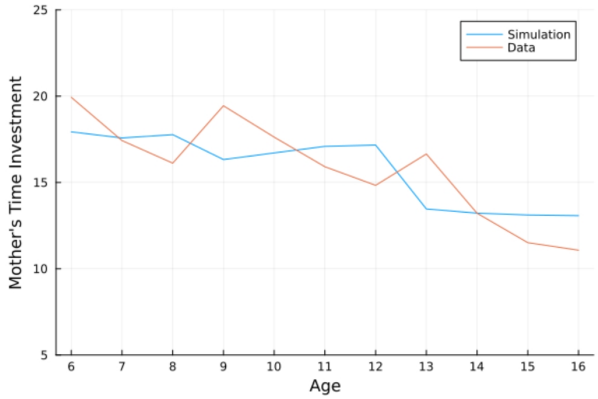
Figure 11 reports the age profiles of latent misbehavior and corporal punishment scheme usage. The model captures the broad level of latent misbehavior, although the data exhibit more age-specific variation, especially at older ages. The model also reproduces the main declining age profile of corporal punishment scheme usage. In the data, scheme usage is highest at younger ages and declines as children become older, and the simulation generates the same qualitative pattern.

Overall, the age profiles show that the model reproduces the main developmental patterns in the data: cognitive capital rises strongly with age, mental stability remains relatively stable, parental time investments decline, child self-investment increases, and corporal punishment scheme usage falls as children grow older. The remaining subsections examine these dimensions more formally using selected targeted moments that were used in the actual estimation procedure.

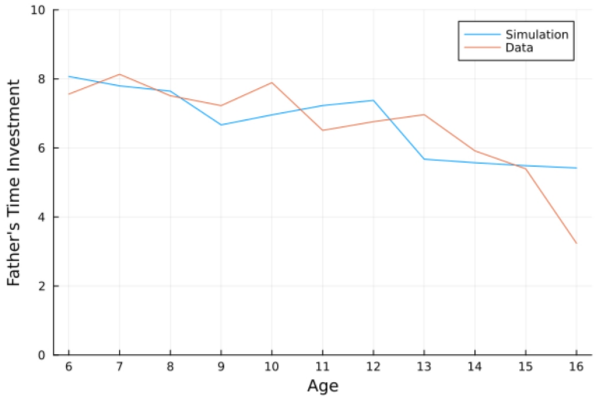
8.2 Cognitive Capital

The model captures the main age profile of cognitive capital, as measured by Letter-Word scores. As shown in Panel A of Table 11, average LW_t rises sharply over the development period. At the level of broader age bins, the average score is 32.02 for children ages 6–8, 45.00 for children ages 9–12, and 49.76 for children ages 13–16. In the simulation, the respective means are 32.40, 44.68, and 50.24.

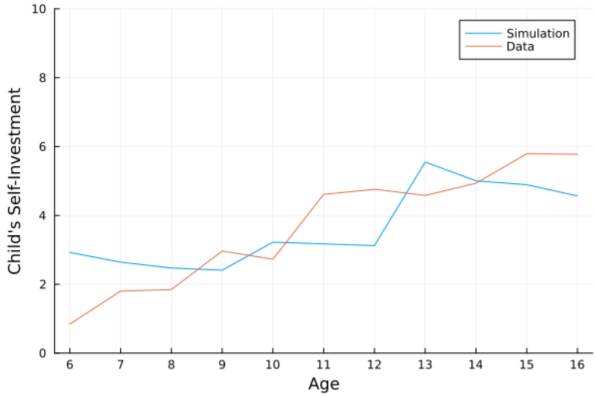
In terms of cognitive capital dispersion, however, the fit is less accurate. The standard deviation of LW_t is 8.93 in the data and 10.50 in the simulation for ages 6–8. For ages 9–12, it is 5.56 in the data and 8.52 in the simulation. For ages 13–16, it is 4.16 in the data and 6.24 in the simulation. Thus, despite matching mean



(a) Mother's time investment

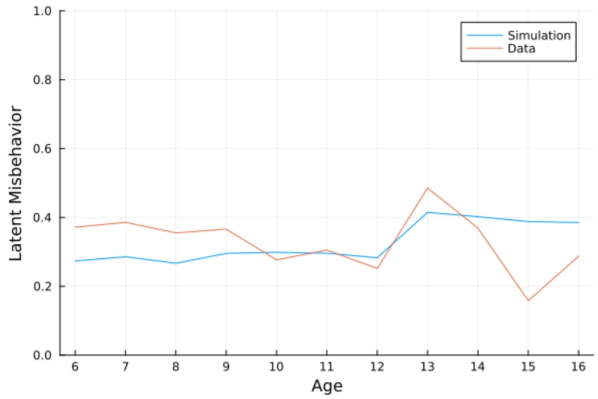


(b) Father's time investment

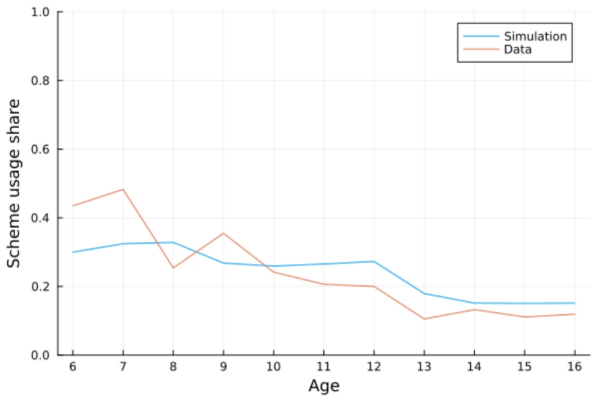


(c) Child self-investment

Figure 10: Model Fit: Time Investments by Child Age



(a) Latent misbehavior



(b) Corporal punishment scheme usage

Figure 11: Model Fit: Misbehavior and Corporal Punishment Scheme Usage by Age

cognitive capital by child's age well, the model produces somewhat higher variation across households than is observed in the data. This discrepancy is especially pronounced at older ages.

The model also matches the negative association between the corporal punishment scheme and cognitive capital. In turn, Panel B of Table 11 presents cognitive capital moments conditional on scheme usage. In particular, for children ages 16 or younger, mean future LW_{t+5} is 47.55 among households that use the scheme and 49.91 among those that do not, with respective simulated means of 45.37 and 50.46. For children older than 8, the corresponding means are 48.74 and 51.06 in the data and 48.66 and 52.56 in the simulation.

Table 11: Selected Fit of Cognitive-Capital Moments

Moment	**Data**	**Simulation**
A. Mean and dispersion of cognitive capital		
$E[LW_t \mid t \leq 8]$	32.018	32.396
$E[LW_t \mid t \in [9, 12]]$	45.004	44.678
$E[LW_t \mid t \in [13, 16]]$	49.759	50.242
$\sigma(LW_t \mid t \leq 8)$	8.927	10.501
$\sigma(LW_t \mid t \in [9, 12])$	5.563	8.518
$\sigma(LW_t \mid t \in [13, 16])$	4.158	6.236
B. Cognitive capital by corporal-punishment scheme usage		
$E[LW_{t+5} \mid t \leq 16, I_{s,t} = 1]$	47.550	45.370
$E[LW_{t+5} \mid t \leq 16, I_{s,t} = 0]$	49.908	50.459
$E[LW_{t+5} \mid t > 8, I_{s,t} = 1]$	48.745	48.658
$E[LW_{t+5} \mid t > 8, I_{s,t} = 0]$	51.059	52.559
$E[LW_t \mid t \leq 16, I_{s,t} = 1]$	37.816	41.140
$E[LW_t \mid t \leq 16, I_{s,t} = 0]$	44.863	45.295

Notes: LW_t denotes the Letter-Word score used to measure cognitive capital. $I_{s,t} = 1$ indicates that the corporal punishment scheme is used in period t.

8.3 Mental Stability

The model fits the mean level of mental stability well across child ages. As reported in Panel A of Table 12, average MS_t is relatively stable across the development period. The mean is 14.39 for children ages 6–8, 13.74 for ages 9–12, and 13.93 for ages 13–16. The corresponding simulated values are 14.11, 13.73, and 13.72. The model therefore reproduces the level and relatively flat age profile of mental stability. However, since the only way household choices affect mental stability is through the corporal punishment scheme, this level of fit is mainly achieved through the flexible TFP specification.

The model also matches the dispersion of mental stability closely. The standard deviation of MS_t is 3.07 in the data and 3.16 in the simulation for ages 6–8. For ages 9–12, it is 3.48 in the data and 3.41 in the simulation. For ages 13–16, it is 3.64 in the data and 3.39 in the simulation.

8.4 Labor Supply and Wages

The fit of labor-force participation is very close. As shown in the upper panel of Table 13, the probability that the mother works is 0.797 in the data for children ages 6–8, 0.811 for ages 9–12, and 0.846 for ages

Table 12: Selected Fit of Mental-Stability Moments

Moment	Data	Simulation
A. Mean and dispersion of mental stability		
$E[MS_t \mid t \leq 8]$	14.389	14.114
$E[MS_t \mid t \in [9, 12]]$	13.735	13.725
$E[MS_t \mid t \in [13, 16]]$	13.934	13.716
$\sigma(MS_t \mid t \leq 8)$	3.067	3.157
$\sigma(MS_t \mid t \in [9, 12])$	3.475	3.408
$\sigma(MS_t \mid t \in [13, 16])$	3.636	3.391

Notes: MS_t denotes mental-stability capital.

13–16. In the simulation, the respective probabilities are 0.806, 0.811, and 0.831. For fathers, participation is very high both in the data and in the estimated model, with little difference between the two. In particular, the observed participation rates for the three age intervals are 0.973, 0.987, and 0.990, while the model produces 0.985, 0.988, and 0.991.

In turn, the middle panel of Table 13 presents mean work hours conditional on positive labor supply. The model matches these moments reasonably well. Mother's average hours in the data are 31.62, 30.41, and 32.93 for the three child-age intervals, respectively. The model gives 29.17, 29.05, and 30.92 hours for mothers in the respective age intervals. Father's average hours are 43.64, 44.52, and 43.78 in the data and 42.11, 43.97, and 46.00 in the simulation. As in the data, fathers in the simulation work substantially more hours than mothers and are close to full-time in most observed periods.

Table 13: Selected Fit of Labor-Supply Moments

	Mother			Father	
Moment	**Data**	**Simulation**	**Moment**	**Data**	**Simulation**
$P(h_{1,t} > 0 \mid t \in [6, 8])$	0.797	0.806	$P(h_{2,t} > 0 \mid t \in [6, 8])$	0.973	0.985
$P(h_{1,t} > 0 \mid t \in [9, 12])$	0.811	0.811	$P(h_{2,t} > 0 \mid t \in [9, 12])$	0.987	0.988
$P(h_{1,t} > 0 \mid t \in [13, 16])$	0.846	0.831	$P(h_{2,t} > 0 \mid t \in [13, 16])$	0.990	0.991
$E[h_{1,t} \mid h_{1,t} > 0, t \in [6, 8]]$	31.616	29.167	$E[h_{2,t} \mid h_{2,t} > 0, t \in [6, 8]]$	43.638	42.108
$E[h_{1,t} \mid h_{1,t} > 0, t \in [9, 12]]$	30.414	29.053	$E[h_{2,t} \mid h_{2,t} > 0, t \in [9, 12]]$	44.519	43.968
$E[h_{1,t} \mid h_{1,t} > 0, t \in [13, 16]]$	32.932	30.921	$E[h_{2,t} \mid h_{2,t} > 0, t \in [13, 16]]$	43.780	45.995
$\sigma(h_{1,t} \mid h_{1,t} > 0)$	12.141	10.670	$\sigma(h_{2,t} \mid h_{2,t} > 0)$	18.632	17.812

Notes: $h_{1,t}$ and $h_{2,t}$ denote mother's and father's weekly hours of market work.

In terms of accepted wages, the model also matches the main patterns by parental age and education. Table 14 shows that wages of both parents increase with age in the data, as they do in the simulation. In particular, mother's mean wage in the youngest and oldest age groups is 13.57 and 22.13, respectively. The corresponding simulated means are 13.06 and 22.83. For father, these means are 20.37 and 32.42 in the data and 18.89 and 33.88 in the simulation.

The main shortcoming in the wage fit is the correlation between spouses' wage offers. The estimated model produces a correlation of 0.881 between $w_{1,t}$ and $w_{2,t}$, against 0.308 observed in the data. Thus, the fit of the marginal wage profiles is reasonably good, but wage offers of parents within the same household are much more strongly correlated in the simulation than in the data.

Table 14: Selected Fit of Wage Moments

	Mother			Father	
Moment	**Data**	**Simulation**	**Moment**	**Data**	**Simulation**
$E[w_{1,t} \mid w_{1,t} > 0, t \in [18, 32]]$	13.568	13.061	$E[w_{2,t} \mid w_{2,t} > 0, t \in [18, 32]]$	20.367	18.894
$E[w_{1,t} \mid w_{1,t} > 0, t \in [33, 37]]$	17.077	15.654	$E[w_{2,t} \mid w_{2,t} > 0, t \in [33, 37]]$	22.668	22.676
$E[w_{1,t} \mid w_{1,t} > 0, t \in [38, 42]]$	18.961	18.464	$E[w_{2,t} \mid w_{2,t} > 0, t \in [38, 42]]$	27.335	26.452
$E[w_{1,t} \mid w_{1,t} > 0, t \in [43, 65]]$	22.126	22.830	$E[w_{2,t} \mid w_{2,t} > 0, t \in [43, 65]]$	32.424	33.876
$\sigma(w_{1,t} \mid w_{1,t} > 0)$	12.141	10.670	$\sigma(w_{2,t} \mid w_{2,t} > 0)$	18.632	17.812
$\text{corr}(w_{1,t}, w_{2,t})$				0.308	0.881

Notes: Wages are observed only for working parents. The simulated moments are computed using the same selection rule.

8.5 Corporal Punishment Scheme Usage

Parental choice of whether to implement the corporal punishment scheme is central to the model. As can be seen from Table 15, in the data parents use the scheme less as the child gets older. Usage is also lower in households with more educated fathers. The estimated model matches the decline with child's age, but the fit by father's education is somewhat less accurate.

For households in which the father has at most 12 years of education, the probability of using the scheme is 0.446 in the data for children ages 6–8, 0.299 for ages 9–12, and 0.158 for ages 13–16. The corresponding simulated probabilities are 0.344, 0.328, and 0.179. For households with fathers who have more than 12 years of education, the empirical probabilities are 0.333, 0.212, and 0.081, compared with simulated probabilities of 0.306, 0.216, and 0.150. Thus, overall, the main qualitative patterns of scheme usage are reproduced, but some differences by father's education are smaller in the model than in the data.

In terms of realized punishment, the model also matches its average intensity conditional on scheme usage. The moment $E[s_t \mid t \leq 16, I_{s,t} = 1]$ is 0.432 in the data and 0.460 in the simulation. Hence, since realized punishment could be positive only when the scheme is used, the model closely matches the average level of realized punishment.

8.6 Time Investments and Non-compliant Leisure

The model fits parental time investment moments reasonably well. Panels A and B of Table 16 report parental and child time investments conditional on positive and zero non-compliant leisure, respectively. Conditional on positive non-compliant leisure, mother's time investment is close to the data in all three age groups. Among children ages 6–8 with positive non-compliant leisure, the data and simulated moments are 16.66 and 17.13. For ages 9–12, they are 16.92 and 17.03, and for ages 13–16 they are 12.94 and 13.57. Father's time investment is also well matched in most cells. Among children ages 6–8 with positive non-compliant leisure, the data moment is 7.75 and the simulated moment is 7.59; among ages 13–16,

Table 15: Selected Fit of Corporal-Punishment Scheme Usage

	$s_2 \leq 12$			$s_2 > 12$	
Moment	**Data**	**Simulation**	**Moment**	**Data**	**Simulation**
$P(I_{s,t} > 0 \mid t \in [6, 8])$	0.446	0.344	$P(I_{s,t} > 0 \mid t \in [6, 8])$	0.333	0.306
$P(I_{s,t} > 0 \mid t \in [9, 12])$	0.299	0.328	$P(I_{s,t} > 0 \mid t \in [9, 12])$	0.212	0.216
$P(I_{s,t} > 0 \mid t \in [13, 16])$	0.158	0.179	$P(I_{s,t} > 0 \mid t \in [13, 16])$	0.081	0.150
$E[s_t \mid t \leq 16, I_{s,t} = 1]$				0.432	0.460

Notes: $I_{s,t} = 1$ denotes use of the corporal punishment scheme. The table reports scheme usage by child age and father's education. The final row reports realized punishment conditional on scheme usage.

the corresponding moments are 5.51 and 5.73. Conditional on zero non-compliant leisure, mother's time investment is 18.67 in the data and 18.23 in the simulation for ages 6–8, 16.82 and 16.68 for ages 9–12, and 13.84 and 12.97 for ages 13–16. Father's time investment is also close: the data and simulated moments are 7.73 and 7.89 for ages 6–8, 6.32 and 6.93 for ages 9–12, and 5.61 and 5.42 for ages 13–16.

The fit of child self-investment is more mixed. The model captures the rise in child self-investment with age, especially at older age intervals. Among children with positive non-compliant leisure, average child self-investment at ages 13–16 is 4.73 in the data and 4.69 in the simulation. Among children with zero non-compliant leisure, the corresponding moments are 5.74 and 5.74. However, the model overpredicts self investment levels at the youngest age interval: for children ages 6–8 with positive non-compliant leisure, child self-investment is 1.44 in the data and 2.88 in the simulation, while for children with zero non-compliant leisure it is 1.68 in the data and 2.35 in the simulation.

In turn, Panel C of Table 16 presents mean latent misbehavior, $E[\xi_t]$, conditional on child's age and scheme usage. As in the data, the estimated model produces higher latent misbehavior in households that use the corporal punishment scheme. In particular, at ages 6–8, mean latent misbehavior in the absence of the scheme is 0.238 in the data and 0.146 in the simulation. Among households using the scheme, the respective means are 0.583 and 0.543. At ages 13–16, these means are 0.321 and 0.383 for households without the scheme and 0.476 and 0.506 for those using it.

The table reports moments of ξ_t, rather than $l_{cd,t}$, because ξ_t is the object observed prior to estimation. Non-compliant leisure $l_{cd,t}$ is constructed from ξ_t using the mapping in Section 5.4, and this mapping depends on an estimated parameter. However, note that there is no such problem with the probabilities of positive non-compliant leisure, $P(l_{cd,t} > 0)$, reported in Panel D of Table 16. Although the level of $l_{cd,t}$ depends on the estimated mapping, the values of latent misbehavior that correspond to positive non-compliant leisure are fixed prior to estimation using the normalization described in Section 5.4. Therefore, the extensive-margin indicator $\mathbf{1}\{l_{cd,t} > 0\}$ is observed before estimation, while the intensive margin $l_{cd,t}$ itself is model-implied.

In the absence of the corporal punishment scheme, the model fits probabilities of positive non-compliant leisure in different samples reasonably well. For ages 6–8, 9–12, and 13–16, the data probabilities are 0.447, 0.426, and 0.529, compared with simulated probabilities of 0.466, 0.484, and 0.540. Conditional on the scheme being used, however, the model underpredicts the probability of positive non-compliant leisure. The empirical probabilities are 0.641, 0.619, and 0.640 across the three age intervals, while the simulated

probabilities are 0.420, 0.452, and 0.521.

Finally, Panel E of Table 16 reports selected second moments. The model captures several second moments of time investment well. The correlation between mother's time and child self-investment is -0.18 in the data and -0.15 in the simulation. The correlation between father's time and child self-investment is -0.14 in the data and -0.13 in the simulation. However, the model overpredicts the correlation between mother's and father's time investments: the data correlation is 0.21, while the simulated correlation is 0.75. The latter could be explained by the unitary household design of the model, in which the mother and father act as one decision-making unit.

9 Counterfactual: Prohibiting the Corporal Punishment Scheme Usage

To study the consequences of banning corporal punishment scheme usage, I conduct the following counterfactual exercise. In the baseline, parents decide whether to use the scheme given its implementation cost $v(t, ed)$. For the counterfactual, this cost is set to $+\infty$. As a result, the scheme is never used, regardless of household characteristics or child's age. This exercise can therefore be viewed as a switch to non-violent parenting. I use the same estimated distribution of preferences, technology parameters, wage process, non-labor income process, and initial conditions in both cases. I compare the baseline and counterfactual in terms of child development and household choices. For child development, I focus on final-period mental stability and cognitive capital. For household choices and resources, I examine child's self-investment, parental labor supply, and household income.

9.1 Parental Response

Table 17 reports parental responses to eliminating the corporal punishment scheme for treated child-periods, defined as periods in which the scheme is used in the baseline among children who are exposed to it at some point. In these periods, median joint parental time investment rises from 20.98 to 25.11 hours when the scheme is banned. As can be seen, parents partly replace the unavailable punishment technology by spending more productive time with the child.

However, parents differ in how they adjust their labor supply. For father, the participation rate is 98.49 percent in the baseline and 98.41 percent in the counterfactual, implying almost no change. Conditional on working, fathers' median work hours even slightly increase, from 45.76 to 46.34. Hence, the labor-supply adjustment is concentrated among mothers. In particular, mother's labor-force participation falls from 82 percent to 79 percent in treated child-periods. Conditional on working, mother's median work hours fall from 30.67 to 27.53. Overall, major reduction in maternal labor supply is not fully compensated by the small increase in father's labor supply, and we observe a decline in median household income, from 1543.14$ to 1496.93$ per week.

Thus, apart from directly changing child's incentives, removal of the corporal punishment scheme also changes how the household allocates its time and resources.

9.2 Effects on Mental Stability

Table 18 reports the effect of eliminating the corporal punishment scheme on final-period mental stability. For children who would otherwise face punishment risk, removal of the scheme mechanically removes its adverse effects from the law of motion for mental stability. Their mental capital therefore increases. However,

Table 16: Selected Fit of Time-Investment and Non-compliant Leisure Moments

Moment	Data	Simulation
A. Time investments: positive non-compliant leisure		
$E[\tau_{1,t} \mid l_{cd,t} > 0, t \in [6,8]]$	16.661	17.125
$E[\tau_{1,t} \mid l_{cd,t} > 0, t \in [9,12]]$	16.921	17.031
$E[\tau_{1,t} \mid l_{cd,t} > 0, t \in [13,16]]$	12.936	13.565
$E[\tau_{2,t} \mid l_{cd,t} > 0, t \in [6,8]]$	7.752	7.593
$E[\tau_{2,t} \mid l_{cd,t} > 0, t \in [9,12]]$	7.906	7.235
$E[\tau_{2,t} \mid l_{cd,t} > 0, t \in [13,16]]$	5.509	5.732
$E[\tau_{c,t} \mid l_{cd,t} > 0, t \in [6,8]]$	1.442	2.878
$E[\tau_{c,t} \mid l_{cd,t} > 0, t \in [9,12]]$	4.176	3.036
$E[\tau_{c,t} \mid l_{cd,t} > 0, t \in [13,16]]$	4.725	4.688
B. Time investments: zero non-compliant leisure		
$E[\tau_{1,t} \mid l_{cd,t} = 0, t \in [6,8]]$	18.668	18.228
$E[\tau_{1,t} \mid l_{cd,t} = 0, t \in [9,12]]$	16.824	16.679
$E[\tau_{1,t} \mid l_{cd,t} = 0, t \in [13,16]]$	13.844	12.972
$E[\tau_{2,t} \mid l_{cd,t} = 0, t \in [6,8]]$	7.726	7.886
$E[\tau_{2,t} \mid l_{cd,t} = 0, t \in [9,12]]$	6.317	6.928
$E[\tau_{2,t} \mid l_{cd,t} = 0, t \in [13,16]]$	5.609	5.415
$E[\tau_{c,t} \mid l_{cd,t} = 0, t \in [6,8]]$	1.676	2.354
$E[\tau_{c,t} \mid l_{cd,t} = 0, t \in [9,12]]$	3.492	2.989
$E[\tau_{c,t} \mid l_{cd,t} = 0, t \in [13,16]]$	5.738	5.739
C. Latent misbehavior		
$E[\xi_t \mid t \in [6,8], I_{s,t} = 0]$	0.238	0.146
$E[\xi_t \mid t \in [9,12], I_{s,t} = 0]$	0.253	0.206
$E[\xi_t \mid t \in [13,16], I_{s,t} = 0]$	0.321	0.383
$E[\xi_t \mid t \in [6,8], I_{s,t} = 1]$	0.583	0.543
$E[\xi_t \mid t \in [9,12], I_{s,t} = 1]$	0.437	0.535
$E[\xi_t \mid t \in [13,16], I_{s,t} = 1]$	0.476	0.506
D. Non-compliant leisure		
$P(l_{cd,t} > 0 \mid t \in [6,8], I_{s,t} = 0)$	0.447	0.466
$P(l_{cd,t} > 0 \mid t \in [9,12], I_{s,t} = 0)$	0.426	0.484
$P(l_{cd,t} > 0 \mid t \in [13,16], I_{s,t} = 0)$	0.529	0.540
$P(l_{cd,t} > 0 \mid t \in [6,8], I_{s,t} = 1)$	0.641	0.420
$P(l_{cd,t} > 0 \mid t \in [9,12], I_{s,t} = 1)$	0.619	0.452
$P(l_{cd,t} > 0 \mid t \in [13,16], I_{s,t} = 1)$	0.640	0.521
E. Miscellaneous second moments		
$\text{corr}(\tau_1, \tau_c)$	-0.18	-0.15
$\text{corr}(\tau_2, \tau_c)$	-0.14	-0.13
$\text{corr}(\tau_1, \tau_2)$	0.21	0.75

Notes: τ_1, τ_2, and τ_c denote mother's time, father's time, and child self-investment time. ξ_t denotes latent misbehavior, while $l_{cd,t}$ denotes non-compliant leisure. $I_{s,t} = 1$ denotes use of the corporal punishment scheme.

Table 17: Parental Response to Eliminating the Corporal Punishment Scheme

	Baseline	**CP Banned**
Model specifications		
Cost of implementing CP scheme $I_{s,t}$	$v(t, ed)$	$+\infty$
Fraction using CP scheme $I_{s,t}$, ages 6–16	0.26	0.00
Median joint parental productive time, $\text{med}(\tau_{1,t} + \tau_{2,t})$		
All children	20.36	21.14
Treated child-periods	20.98	25.11
Father's labor participation rate, $P(h_{2,t} > 0)$		
All children	98.80%	98.78%
Treated child-periods	98.49%	98.41%
Median father's work hours, $\text{med}(h_{2,t} \mid h_{2,t} > 0)$		
All children	46.41	46.56
Treated child-periods	45.76	46.34
Mother's labor participation rate, $P(h_{1,t} > 0)$		
All households	81%	80%
Treated child-periods	82%	79%
Median mother's work hours, $\text{med}(h_{1,t} \mid h_{1,t} > 0)$		
All households	28.50	27.64
Treated child-periods	30.67	27.53
Median household weekly income, $\text{med}(Y_t)$		
All households	1673.78	1659.50
Treated child-periods	1543.14	1496.93

Notes: "Baseline" corresponds to the estimated baseline model. "CP Banned" corresponds to the counterfactual in which the cost of using the corporal punishment scheme is set to $+\infty$. Treated child-periods are periods in which the corporal punishment scheme is used in the baseline model among children who are exposed to the scheme at some point. $h_{1,t}$ and $h_{2,t}$ denote mother's and father's market work hours, respectively. Y_t denotes weekly household income.

the increase in average final-period mental capital across all households comes from children directly exposed to the scheme at some point during childhood. For these children, the mean rises from 3.43 to 3.70. The effect is even larger for children who both face the scheme and have a positive expected punishment at some age. Among these children, mental capital in the final period rises from 3.10 in the baseline to 3.57 under the ban.

Table 18: Impact of Switch to Non-Violent Parenting: Mental Stability

	Baseline	**CP Banned**
Model specifications		
Cost of implementing CP scheme $I_{s,t}$	$v(t, ed)$	$+\infty$
Fraction using CP scheme $I_{s,t}$, ages 6–16	0.26	0.00
Final-period mental capital, $E[\log(k_{p,i}(t_{i,end}))]$		
All children	3.61	3.70
		$(+0.03\sigma)$
Children s.t. CP scheme used at some age,	3.43	3.70
$I_{s,t} = 1$ for some t		$(+0.10\sigma)$
Children s.t. CP scheme used at some age,	3.10	3.57
positive punishment expectation,		$(+0.18\sigma)$
$I_{s,t} = 1$ and $E[s_t] > 0$ for some t		

Notes: "Baseline" corresponds to the estimated baseline model. "CP Banned" corresponds to the counterfactual in which the cost of using the corporal punishment scheme is set to $+\infty$. Parentheses report changes in units of the baseline standard deviation of final-period mental capital.

9.3 Effects on Cognitive Capital

The effect of eliminating the corporal punishment scheme on cognitive capital is more ambiguous. Table 19 shows that the average effect on final-period cognitive capital is close to zero and slightly negative for all children. Mean final-period cognitive capital decreases from 3.99 in the baseline model to 3.97 when the scheme is banned. Among children who are exposed to the scheme at some age, the average falls slightly more, from 3.04 to 2.97.

Despite the small average change, the effect differs substantially between children. Among children who experience a positive final period cognitive-capital change, final-period cognitive capital rises from 3.54 to 3.75. Among children who experience a negative final period cognitive-capital change, it falls from 2.77 to 2.56. The share of children with a positive cognitive-capital change is 35.58 percent. Thus, eliminating the scheme improves cognitive outcomes for around one third of exposed children and worsens them for the rest.

The ambiguous cognitive-capital effect reflects several opposing mechanisms, as described in Table 20. On the positive side, eliminating the scheme removes the direct negative effect of the scheme on cognitive total factor productivity. It also raises mental stability, which can spill over into cognitive capital, and induces parents to increase time investments in order to prevent misbehavior without relying on punishment.

However, removing the scheme could also have negative effects. First, obviously it directly increases incentive to misbehave. Although the technological effects of increased misbehavior on cognitive skills was estimated to be negligible as discussed in Section 7.2, it also affects cognitive capital indirectly in three

Table 19: Impact of Switch to Non-Violent Parenting: Cognitive Skills

	Baseline	**CP Banned**
Model specifications		
Cost of implementing CP scheme $I_{s,t}$	$v(t, ed)$	$+\infty$
Fraction using CP scheme $I_{s,t}$, ages 6–16	0.26	0.00
Final-period cognitive capital, $E[\log(k_{h,i}(t_{i,end}))]$		
All children	3.99	3.97
		(-0.01σ)
Children s.t. CP scheme used at some age,	3.04	2.97
$I_{s,t} = 1$ for some t		(-0.03σ)
Share of children with positive change	–	35.58%
Children with positive change,	3.54	3.75
$\Delta_{I_s} \log(k_{h,i}(t_{i,end})) > 0$		$(+0.10\sigma)$
Children with negative change,	2.77	2.56
$\Delta_{I_s} \log(k_{h,i}(t_{i,end})) < 0$		(-0.10σ)

Notes: "Baseline" corresponds to the estimated baseline model. "CP Banned" corresponds to the counterfactual in which the cost of using the corporal punishment scheme is set to $+\infty$. Parentheses report changes in units of the baseline standard deviation of final-period cognitive capital.

channels. First, increase in relative attractiveness of misbehavior pushes children to decrease self-investment. Second, parents respond by increasing productive time with the child in order to avert misbehavior. Since the child's time is limited, this increase in parental time decreases self-investment even further. Third, the increase in productive time with parents crowds out parental work hours. Lower labor supply reduces household resources available for monetary investments in the child.

Thus, the net cognitive effect depends on which set of forces dominates for a given child. In the next subsection we focus on the drivers of cognitive outcome response heterogeneity in more detail.

Table 20: Mechanisms Behind the Cognitive-Capital Response

Positive effects on cognitive capital	**Negative effects on cognitive capital**
No direct harm to cognitive skills through TFP, $R_{h,t} \uparrow$.	More incentive to misbehave, $l_{cd,t} \uparrow$.
Parents invest more time to avert misbehavior, $\tau_{1,t} + \tau_{2,t} \uparrow$.	Less incentive to study, $\tau_{c,t} \downarrow$.
Positive spillovers from higher mental stability, $k_{p,t} \uparrow$.	Lower monetary investment into the child's capital, $e_{c,t} \downarrow$.

9.4 Dynamics by Child's Preference Group

The previous results suggest that the cognitive effect of banning the scheme is highly heterogeneous and depends on household's response. To examine the mechanism more directly, I split children who are affected by corporal punishment at some point during their upbringing into two groups. The first group has relatively

low preference for non-compliant leisure and relatively high preference for cognitive capital among the treated children:

$$\lambda_2 \leq \text{med}(\lambda_2), \qquad \lambda_3 > \text{med}(\lambda_3).$$

I refer to this group as children with low misbehavior taste and high self-investment valuation. The second group has relatively high preference for non-compliant leisure and relatively low preference for cognitive capital among the treated children:

$$\lambda_2 > \text{med}(\lambda_2), \qquad \lambda_3 \leq \text{med}(\lambda_3).$$

I refer to this group as children with high misbehavior taste and low self-investment valuation.

Figure 12 plots the age profile of the change in next-period cognitive capital for these two groups. For children with low misbehavior taste and high cognitive-capital valuation, banning the scheme raises next-period cognitive capital at every age. Moreover, the positive cumulative effect increases as children grow older. In turn, children with high misbehavior taste and low cognitive-capital valuation experience a decrease at every age, with larger losses later in the development period. Thus, whether the ban raises or lowers cognitive capital depends on child's preference type, with opposite effects for the two groups.

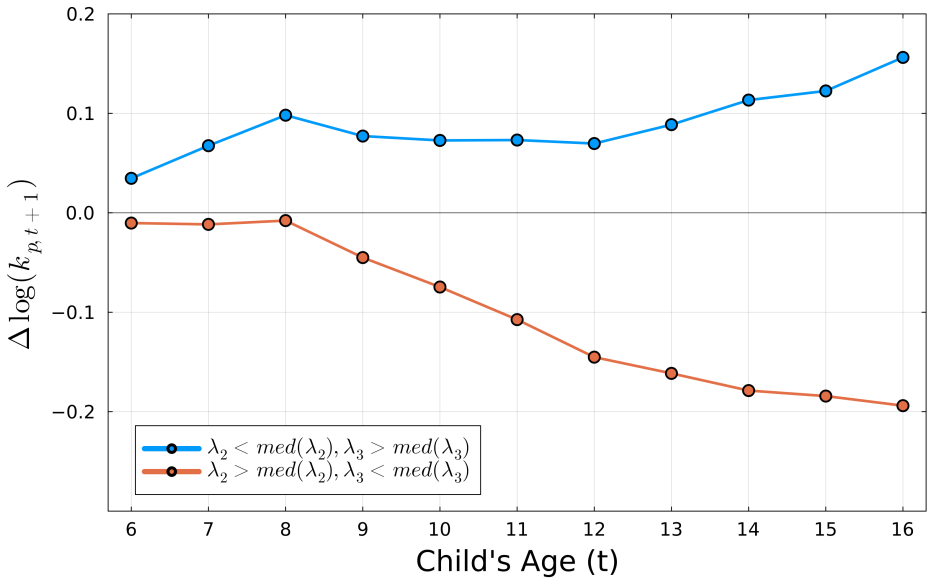


Figure 12: Counterfactual Effect on Next-Period Cognitive Capital by Preference Type

Notes: The figure reports the change in next-period cognitive capital when the cost of using the corporal punishment scheme is set to $+\infty$. The first group satisfies $\lambda_2 \leq \text{med}(\lambda_2)$ and $\lambda_3 > \text{med}(\lambda_3)$. The second group satisfies $\lambda_2 > \text{med}(\lambda_2)$ and $\lambda_3 \leq \text{med}(\lambda_3)$.

To understand why the cognitive response differs across these two groups, Table 21 reports their respective average choice responses. The first margin is joint parental time. For children with low misbehavior taste and high self-investment valuation, the ban leads to a large parental response. Joint time with parents increases from 19.72 to 27.49. For children with high misbehavior taste and low self-investment valuation,

the corresponding increase is negligible: joint parental time is 36.04 in the baseline and 36.17 under the ban. However, this smaller response does not mean that these parents are less involved. In fact, they are already spending considerably more time with the child in the baseline. Hence, after removal of the scheme, these parents have less leeway to increase time with the child as an alternative mean of discipline.

Table 21: Choice Responses to Eliminating the Corporal Punishment Scheme by Preference Type

	Low λ_2, High λ_3			**High λ_2, Low λ_3**		
Average over ages 6–8	**Baseline**	**CP banned**	**Δ**	**Baseline**	**CP banned**	**Δ**
Joint parental time, $\tau_{1,t} + \tau_{2,t}$	19.72	27.49	+7.77	36.04	36.17	+0.13
Misbehavior, $l_{cd,t}$	9.95	43.05	+33.10	35.63	40.74	+5.11
Child self-investment, $\tau_{c,t}$	4.51	0.48	−4.03	0.17	0.01	−0.16
Monetary investment, e_t	514.04	501.14	−12.90	498.38	498.10	−0.28

Notes: The table reports average choice levels over ages 6–8. Δ is defined as the counterfactual value under the ban minus the baseline value. The first group satisfies $\lambda_2 \leq \text{med}(\lambda_2)$ and $\lambda_3 > \text{med}(\lambda_3)$. The second group satisfies $\lambda_2 > \text{med}(\lambda_2)$ and $\lambda_3 \leq \text{med}(\lambda_3)$.

The next margin is child's self-investment. Overall, the difference in child response within this dimension is spurred by the differences in preferences and parental adjustment. In the low-misbehavior-taste group, it remains moderately positive, although heavily declined from 4.51 to 0.48. Thus, in this group parents replace part of child's independent self-investment with joint parental time, and the child continues to moderately invest on her own. As a result, more productive time with parents partly compensates for the reduction in self-investment and helps maintain cognitive capital accumulation.

For children with high misbehavior taste and low self-investment valuation, the situation is different. Their baseline misbehavior is already high, their baseline self-investment is already extremely low, and their parents are already spending a large amount of time with them. When the scheme is banned, these parents lose an additional instrument for controlling behavior, but cannot do much more by increasing time with the child. At the same time, the child's self-investment moves essentially to zero. As a result, these children lose the disciplining effect of the scheme, and there is no compensating increase in productive inputs to offset it. Cognitive capital therefore decreases.

Figure 13 then plots the analogous age profile for mental capital change. In contrast to cognitive capital, eliminating the scheme raises mental stability for both groups. However, we still observe substantial heterogeneity which is this time in favor of high-misbehavior-taste group.

To explain it, note that the children with high taste for misbehavior and low valuation of cognitive capital are more likely to choose very high levels of misbehavior under the scheme and therefore face high expected punishment in the baseline specification. Removing the scheme eliminates this punishment channel, generating sizable mental-stability gains. In turn, children with low preference for non-compliant leisure and high preference for cognitive capital often choose a corner solution with zero misbehavior in the presence of the scheme. In these cases, punishment cannot be realized. Since, as discussed in Section 7, the main source of harm for mental stability at younger ages is realized punishment, and since the direct TFP effect of the

scheme itself is negligible during those ages, the mental-stability gains for this group are close to zero in early childhood. Their gains become sizable mainly in adolescence, when the direct effect of the scheme on mental-stability TFP becomes considerable.

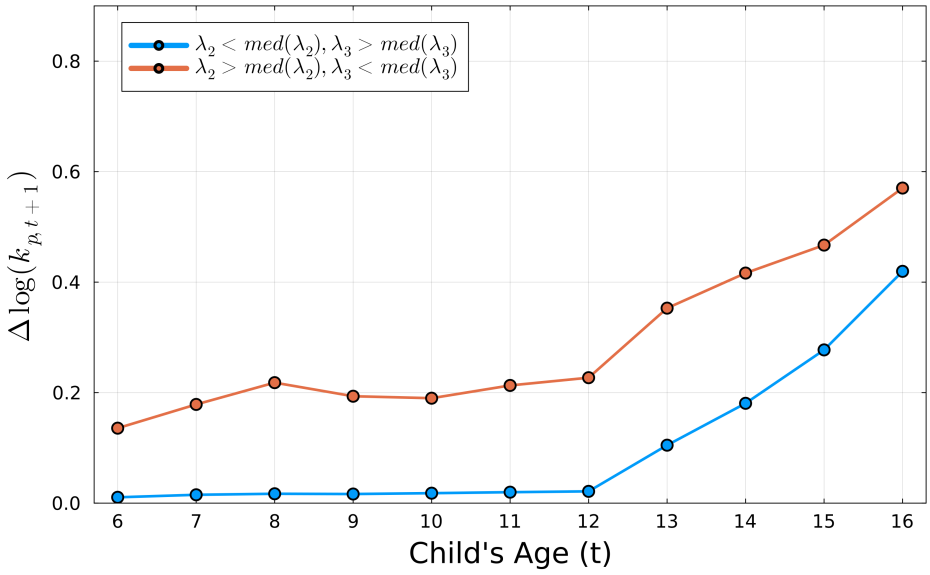


Figure 13: Counterfactual Effect on Next-Period Mental Capital by Preference Type

Notes: The figure reports the change in next-period mental capital when the cost of using the corporal punishment scheme is set to $+\infty$. The first group satisfies $\lambda_2 \leq \text{med}(\lambda_2)$ and $\lambda_3 > \text{med}(\lambda_3)$. The second group satisfies $\lambda_2 > \text{med}(\lambda_2)$ and $\lambda_3 \leq \text{med}(\lambda_3)$.

These dynamic results imply that differences in the counterfactual effects by child's preferences have different explanations for cognitive capital and mental stability. In terms of cognitive capital, what matters is how much the household can replace the scheme with other productive inputs. Parents in the low-misbehavior-taste group increase joint time with the child considerably after the ban. Although child's self-investment decreases, more time with parents partly offsets this loss. This, together with the positive TFP effect of removing the scheme, leads to an overall increase in cognitive capital. In turn, parents in the high-misbehavior-taste group already spend a large amount of time with the child in the baseline and increase it little in response to the ban. Child's self-investment is already very low and decreases to almost zero. For this group, therefore, removal of the disciplining instrument is not accompanied by an increase in productive inputs sufficient to compensate for its loss.

For mental stability, in turn, the main source of heterogeneity is the amount of expected punishment in the baseline. When the scheme is available, children with high taste for non-compliant leisure are more likely to misbehave a lot and hence expect more punishment. Thus, for these children, the ban eliminates expected punishment and leads to a substantial improvement in mental stability. On the other hand, children with low taste for non-compliant leisure often choose zero or small amounts of misbehavior under the scheme. As a result, they face much lower expected punishment in the baseline, and the improvement in their mental stability is moderate. This is especially so at younger ages because the scheme's direct negative effect on

mental stability is small at those ages.

9.5 Change in Parental Welfare by Child's Preference Group

The heterogeneous child responses also translate into different welfare dynamics for parents. Figure 14 plots the change in parental welfare when the cost of using the corporal punishment scheme is set to $+\infty$, separately for the two preference-type groups. As with cognitive capital, two aspects of the response matter for parental welfare. These are how much parents have to adjust immediately after the ban and how child capital evolves afterwards.

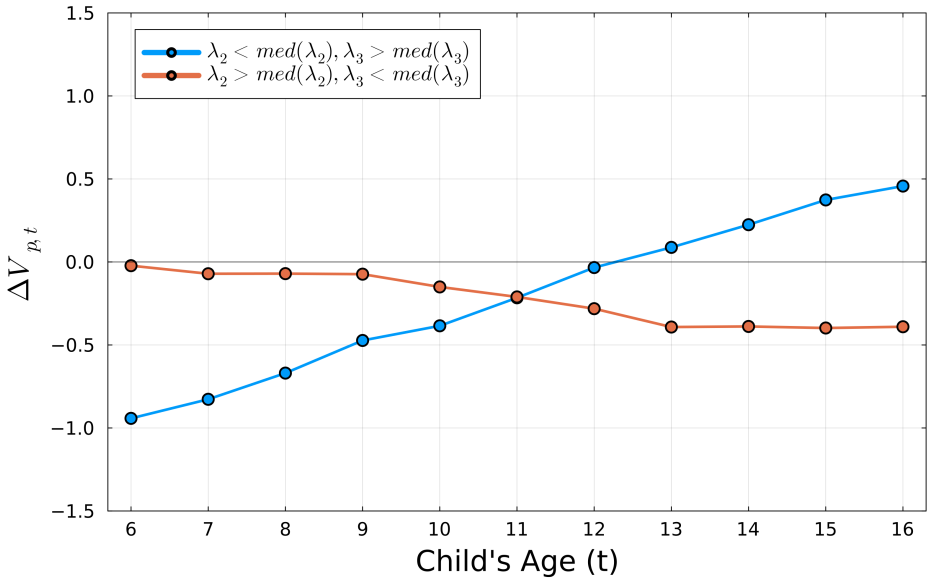


Figure 14: Counterfactual Effect on Parental Welfare by Preference Type

Notes: The figure reports the change in parental welfare when the cost of using the corporal punishment scheme is set to $+\infty$. The first group satisfies $\lambda_2 \leq \text{med}(\lambda_2)$ and $\lambda_3 > \text{med}(\lambda_3)$. The second group satisfies $\lambda_2 > \text{med}(\lambda_2)$ and $\lambda_3 \leq \text{med}(\lambda_3)$.

For the group with low misbehavior taste and high valuation of cognitive capital, parental welfare decreases immediately after the ban. As can be seen from Table 21, the adjustment requires a sharp increase in joint parental time. It is also associated with more child misbehavior and less self-investment. The initial decline in welfare comes from these direct adjustment costs. Parents allocate substantially more time to managing the child and consequently have less time and resources for other uses. They also get greater disutility from misbehavior. This initial loss then disappears. With the accumulation of child capital under the counterfactual, parental welfare eventually exceeds its baseline level. Thus, parents in this group bear a large cost of behavioral adjustment in the short run and subsequently benefit from child capital accumulation.

The welfare dynamics are different for parents of children with high misbehavior taste and low valuation of cognitive capital. For this group, the initial welfare response is close to zero. The reason is that little adjustment takes place when the scheme is banned. Parents in this group already spend a large amount of time with the child in the baseline, and the counterfactual increase in joint parental time is negligible. The decrease in child's self-investment to almost zero affects capital with a delay, so that its effect on utility is

discounted. Misbehavior, in turn, increases only moderately from an already high base. Over time, however, as negative changes in cognitive capital accumulate, the welfare difference becomes more negative.

Thus, in the low-misbehavior-taste group, parents pay a large cost of adjustment early on and benefit later from improvements in child capital. For the high-misbehavior-taste group, there is little initial cost of adjustment, but the decline in cognitive capital leads to parental welfare losses later.

9.6 Summary of the Counterfactual Results and Discussion

The counterfactual results have three main implications. First, I find that prohibiting usage of the corporal punishment scheme improves child's mental stability. Recall that setting the implementation cost to $+\infty$ removes the expected-punishment channel from the formation of mental-stability capital. As a result, children who face positive punishment risk benefit the most. Second, the implications for cognitive capital are ambiguous. I find almost no change in average final-period cognitive capital, with an increase for approximately one third of exposed children and a decrease for the rest. Main source of ambiguity comes from different equilibrium adjustments. In particular, although the estimated direct effect of scheme usage is strongly negative, banning it also forces households to adjust their allocation of time and resources. In some households, parents replace the scheme with higher joint time investments, which help sustain cognitive accumulation. In others, parents already spend substantial time with the child and cannot offer much more. Thus, these households lose a disciplining option without a compensating increase in productive inputs, which leads to capital deterioration. Third, an important and novel implication of the model is that the switch to non-violent parenting affects parental labor supply. In treated child-periods, mothers' labor participation falls from 82 to 79 percent, and their median work hours decrease from 30.67 to 27.53. This is accompanied by a decrease in weekly household income from 1543.14 to 1496.93 dollars. Hence, the ban changes not only child's incentives and outcomes, but also parental time allocation, with mothers in particular giving up work in order to spend more time with the child.

Notice, however, that in my model parents can use a "stick"—the corporal punishment scheme—but have no explicit "carrot". In particular, once the scheme is banned, they cannot directly reward child self-investment. For instance, such an option is available in DFVW (2025), where parents can choose whether to implement a conditional allowance scheme. In my model, on the other hand, parents can only increase productive time with the child as a non-violent alternative to punishment. On the bright side, this allows me to isolate the trade-off between corporal punishment and parental time. However, it could also lead to an overestimation of the decrease in self-investment for households that could in reality replace punishment with rewards. The counterfactual therefore describes removal of physical punishment given the available alternative disciplinary technology.

At the same time, treating productive parental time as the main non-violent alternative could be viewed as a plausible approximation. In particular, most other methods of discipline, such as, for example, imposing chores and restricting privileges, require additional enforcing and monitoring time on the side of parents. The additional joint productive time in the model could therefore stand for these non-violent forms of behavioral management. The trade-off in the model would then describe parents losing a punishment instrument which requires little time and having to discipline their child through more time-intensive interactions instead.

10 Conclusion

In this paper, I build and estimate a dynamic model of child development to study the impact of corporal punishment availability on child's development and household resource allocation. Comparing children who are punished with those who are not, as was many times done in reduced form before, is not enough to evaluate the consequences of punishment in such a setting. Parents use corporal punishment as one of their disciplining options, and removing this option forces them to adjust their parenting choices, which could in turn change child's behavior, parental inputs, labor supply and household income.

In my model parents can discourage misbehavior either by tightening child's time budget through spending more time together or by implementing a corporal punishment scheme. The latter directly discourages misbehavior and indirectly incentivizes child's self-investment, but both implementation of the scheme and realized punishment could hurt child's human capital. In turn, the child acts as an active decision-maker and chooses self-investment, regular leisure and misbehavior given parental choices. This allows me to account for equilibrium household responses to punishment removal along with its direct developmental effects.

Given the estimated model, I study a counterfactual in which the corporal punishment scheme is made prohibitively costly. I find that banning the scheme improves child's mental well-being, especially for children who face positive expected punishment in the baseline, since the ban directly removes this punishment channel. The effect on cognitive capital, however, is ambiguous. Although its average changes little, this masks substantial heterogeneity in household outcomes. In particular, cognitive capital increases for children whose parents replace the scheme with more joint time. For children whose parents are either incapable or unwilling to do that, the effect is negative.

An important contribution of the counterfactual analysis is that it accounts for parental labor-supply adjustments. Replacing the scheme requires more parental involvement, which changes the opportunity cost of parenting and leads to lower maternal labor-force participation and work hours, with almost no change in fathers' labor supply. Household income falls as well.

The parental welfare response to the ban also depends on child's preferences and household's ability to substitute toward non-violent supervision. For parents of children with low taste for misbehavior, the ban initially leads to a welfare loss, since they have to reallocate substantial time toward the child while also facing more misbehavior and lower child self-investment. However, this loss is temporary, and parents benefit later as child capital accumulates. For parents of children with high taste for misbehavior, the initial welfare change is small because little adjustment takes place immediately after the ban. Over time, their welfare falls as cognitive capital deteriorates.

Notice, however, that my model allows parents to use a stick, the corporal punishment scheme, but has no explicit carrot: parents cannot replace the scheme with allowances, privileges or other incentive contracts to directly reward self-investment. Their main non-violent alternative is productive time with the child. On the bright side, this allows me to isolate the trade-off between punishment and parental time. However, it could also overstate the decline in self-investment for households that could in reality use reward-based discipline. At the same time, treating parental time as a proxy for non-violent discipline could be viewed as a plausible approximation. In particular, enforcing rules, restricting privileges and following through on consequences require additional time and involvement on the side of parents. The time input in the model could therefore

stand for a broader set of time-intensive non-violent disciplining methods.

References

Agostinelli, F., Doepke, M., Sorrenti, G., and Zilibotti, F. (2020). It takes a village: The economics of parenting with neighborhood and peer effects. NBER Working Paper No. 27050.

Cunha, F., and Heckman, J. J. (2007). The technology of skill formation. *American Economic Review*, 97(2), 31–47.

Cunha, F., Heckman, J. J., and Schennach, S. M. (2010). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica*, 78(3), 883–931.

Cunha, F., Nielsen, E., and Williams, B. (2021). The econometrics of early childhood human capital and investments. *Annual Review of Economics*, 13, 487–513.

Del Boca, D., and Flinn, C. (2009). Endogenous household interaction. IZA Discussion Paper No. 4377.

Del Boca, D., Flinn, C., and Wiswall, M. (2014). Household choices and child development. *Review of Economic Studies*, 81(1), 137–185.

Del Boca, D., Flinn, C., Verriest, E., and Wiswall, M. (2025). Parenting with patience: Parental incentives and child development. Manuscript.

Doepke, M., and Zilibotti, F. (2017). Parenting with style: Altruism and paternalism in intergenerational preference transmission. *Econometrica*, 85(5), 1331–1371.

Doepke, M., and Zilibotti, F. (2019). *Love, Money, and Parenting: How Economics Explains the Way We Raise Our Kids*. Princeton University Press.

Gershoff, E. T. (2002). Corporal punishment by parents and associated child behaviors and experiences: A meta-analytic and theoretical review. *Psychological Bulletin*, 128(4), 539–579.

Gershoff, E. T., and Grogan-Kaylor, A. (2016). Spanking and child outcomes: Old controversies and new meta-analyses. *Journal of Family Psychology*, 30(4), 453–469.

Heilmann, A., Mehay, A., Watt, R. G., Kelly, Y., Durrant, J. E., van Turnhout, J., and Gershoff, E. T. (2021). Physical punishment and child outcomes: A narrative review of prospective studies. *The Lancet*, 398(10297), 355–364.

MacKenzie, M. J., Nicklas, E., Waldfogel, J., and Brooks-Gunn, J. (2012). Corporal punishment and child behavioural and cognitive outcomes through 5 years of age: Evidence from a contemporary urban birth cohort study. *Infant and Child Development*, 21(1), 3–33.

MacKenzie, M. J., Nicklas, E., Waldfogel, J., and Brooks-Gunn, J. (2013). Spanking and child development across the first decade of life. *Pediatrics*, 132(5), e1118–e1125.

McGuire, A., and Jackson, Y. (2018). A multilevel meta-analysis on academic achievement among maltreated youth. *Clinical Child and Family Psychology Review*, 21(4), 450–465.

McLaughlin, K. A., Sheridan, M. A., and Lambert, H. K. (2014). Childhood adversity and neural development: Deprivation and threat as distinct dimensions of early experience. *Neuroscience and Biobehavioral Reviews*, 47, 578–591.

Sheridan, M. A., and McLaughlin, K. A. (2014). Dimensions of early experience and neural development: Deprivation and threat. *Trends in Cognitive Sciences*, 18(11), 580–585.

Verriest, E. (2025). Household labor supply, child development and childcare policies in the United States.

Appendix

A Econometric Specification

In this section, I describe econometric specification used in identification and estimation of the model. Overall, for each household there are four sets of unknown parameters: 1) preference parameters as specified in (1) and (2); 2) corporal punishment scheme parameters as specified in (3), (4) and (5); 3) technology parameters, which govern the evolution of capital as specified in (6) and (7); 4) income process parameters, which govern the distribution of wages and non-labor income in (10).

A.1 Preferences

I assume that preferences are randomly heterogeneous across households but fixed over time. I allow for correlations both within and across the draws of child and parental preference parameters. Overall, each household has 10 preference coefficients entering the parental $(a_1, \ldots, a_6)$ and child's $(\lambda_1, \ldots, \lambda_4)$ utility functions. In particular, each household draws its preference vector according to the following scheme:

1. Draw an 8-dimensional latent vector

$$x = (x_1, \ldots, x_8) \sim N(\mu, \Omega). \tag{25}$$

2. Map the draw to preference parameters as follows:

$$\alpha_i = \frac{\exp(x_i)}{1 + \sum_{j=1}^{5} \exp(x_j)}, \quad i = 1, 2, 3, 4, 6. \qquad \alpha_5 = \frac{1}{1 + \sum_{j=1}^{5} \exp(x_j)}.$$

$$\lambda_k = \frac{\exp(x_{5+k})}{1 + \sum_{j=1}^{3} \exp(x_{5+j})}, \quad k = 1, 2, 3. \qquad \lambda_4 = \frac{1}{1 + \sum_{j=1}^{3} \exp(x_{5+j})}.$$

For identification purposes, one preference coefficient has been normalized in each of the parental and child's utility functions. For consistency, the normalized coefficient in both cases corresponds to the preference for mental stability $k_{p,t}$.

Randomly heterogeneous preferences have several advantages over homogeneous preferences in the context of the model. First, they help explain persistent behavioral differences across households in terms of parenting style and child's time allocation choices. This allows to better fit household conditional choice moments, such as for example average self-investment in a presence of corporal punishment scheme, which are crucial for identification of some of the model parameters. Second, they allow for richer substitution patterns in response to counterfactuals of interest, such as corporal punishment scheme ban. With homogeneous preferences, variation in responses to counterfactuals is created only by the set of household state variables (e.g., age, education, income) which I found to be not rich enough.

A.2 Corporal Punishment Scheme Parameters

In this section, I describe the econometric specification of the corporal punishment scheme parameters featured in equations (3), (4) and (5). The specification of the parameters capturing the effect of the scheme on capital evolution is described in the next subsection, together with the other technology parameters.

Parameter b, which governs the Poisson process of realized punishment in equation (4), and parameter

δ, which captures the child's disutility from one unit of punishment, are both assumed to be non-negative, time-invariant, and homogeneous across households.

To capture the age and education trends in scheme usage, parameter v_t in equation (3), which governs the parental utility cost of using the scheme, is allowed to vary with the child's age and the father's education, ed_2. Specifically, I assume that v_t is piecewise constant across three age bins, with bin-specific coefficients:

$$v_t = \exp\left(v_{0,g(t)} + v_{1,g(t)} \cdot ed_2\right), \tag{26}$$

where $g(t) \in \{1, 2, 3\}$ indexes the child's age bin at time t, corresponding to ages 6–8, 9–12, and above 12, respectively. Thus, the model includes three pairs of parameters, $(v_{0,1}, v_{1,1})$, $(v_{0,2}, v_{1,2})$, and $(v_{0,3}, v_{1,3})$, one for each age bin.

A.3 Technology Parameters and Total Factor Productivity

In this section, I describe the specification of the technology parameters governing the child's capital accumulation laws given by equations (6) and (7), including the corresponding total factor productivity components. Technology parameters are allowed to vary with time but are fixed across households. Although my identification strategy potentially allows for period-specific technology coefficients, I impose a more parsimonious specification for most parameters and assume that they are piecewise constant across three age bins. Specifically, the three bins correspond to ages 6–8, 9–12, and above 12. I use this specification for two reasons. First, these three bins divide the sample into roughly equal parts. Second, they correspond to three economically meaningful developmental stages: childhood, pre-teen years, and adolescence. Total factor productivity is introduced in both laws of motion to capture age-dependent shifts in capital accumulation that are not directly accounted for by the observed inputs.

All technology parameters are restricted to be non-negative, except for the effect of the corporal punishment scheme on total factor productivity, whose sign is left unrestricted. Additionally, let $g(t) \in \{1, 2, 3\}$ index the child's age bin at time t, corresponding to ages 6–8, 9–12, and above 12, respectively

For cognitive skills, total factor productivity is parameterized as

$$R_{h,t}(1_{s,t}) = d_1 + \frac{d_2 - d_1}{1 + \exp(-d_3(t - d_4))} + \rho_{h,0,g(t)}\, 1_{s,t}, \tag{27}$$

where the first two terms capture the baseline age profile of cognitive-skill productivity and the third term captures the effect of the corporal punishment scheme. Here, $d_1, d_2, d_3, d_4 \geq 0$ are time-invariant, and impact of the scheme on TFP, $\rho_{h,0,g(t)}$, is assumed to be piecewise constant across the three age bins.

The remaining parameters in the cognitive-skills law of motion given by (6), $\gamma_{1,t}, \ldots, \gamma_{7,t}$, are assumed to be non-negative. The parameters on parental time investments, $\gamma_{1,t}$ and $\gamma_{2,t}$, are additionally allowed to depend on, respectively, the mother's and the father's educational attainment. Specifically, I assume that

$$\gamma_{1,t} = \exp\left(\gamma_{1,0,g(t)} + \gamma_{1,1,g(t)} \cdot ed_1\right), \tag{28}$$

and

$$\gamma_{2,t} = \exp\left(\gamma_{2,0,g(t)} + \gamma_{2,1,g(t)} \cdot ed_2\right). \tag{29}$$

Thus, the model includes three pairs of parameters for $\gamma_{1,t}$ and three pairs of parameters for $\gamma_{2,t}$, one for each age bin.

The remaining cognitive-skill technology parameters, $\gamma_{3,t}, \gamma_{4,t}, \gamma_{5,t}, \gamma_{6,t}, \gamma_{7,t}$, are assumed to be piecewise constant across the same three age bins. Thus, for each $j \in \{3, 4, 5, 6, 7\}$,

$$\gamma_{j,t} = \exp(\gamma_{j,0,g(t)}). \tag{30}$$

For mental stability, total factor productivity is parameterized as

$$R_{p,t}(1_{s,t}) = p_t + \rho_{p,0,g(t)}\, 1_{s,t}, \tag{31}$$

where $p_t \geq 0$ is allowed to vary freely over time. This period-specific intercept is included because variation in mental stability observed in the data cannot be explained by variation in choice variables $1_{s,t}$ and s_t alone. The term $\rho_{p,0,g(t)}$ captures the effect of the corporal punishment scheme on mental stability productivity. As in the cognitive skills equation, the sign of $\rho_{p,0,g(t)}$ is left unrestricted.

The remaining parameters in the mental-stability law of motion (7), $\delta_{1,t}$ and $\delta_{2,t}$, are assumed to be non-negative and piecewise constant across the same three age bins:

$$\delta_{m,t} = \exp(\delta_{m,0,g(t)}), \qquad m \in \{1, 2\}. \tag{32}$$

A.4 Wage Offers and Non-Labor Income

In this section, I describe the econometric specification of wage offers and non-labor income, which enter the household budget constraint as exogenous state variables. The specification is identical to DFVW (2025). Both processes are allowed to vary over time and across households as functions of parental age and educational attainment.

The hourly wage offer process for each parent $i \in \{1, 2\}$ is specified as

$$\ln w_{i,t} = \eta_{0,i} + \eta_{1,i} \cdot age_{i,t} + \eta_{2,i} \cdot ed_i + \varepsilon_{i,t}, \tag{33}$$

where $age_{i,t}$ and ed_i denote, respectively, the age and educational attainment of parent i in period t. The wage shocks are assumed to be serially uncorrelated and jointly normally distributed:

$$(\varepsilon_{1,t}, \varepsilon_{2,t}) \sim N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\sigma^2_{w1} & \sigma_{w1,w2}\\ \sigma_{w1,w2} & \sigma^2_{w2}\end{bmatrix}\right). \tag{34}$$

This specification allows for contemporaneous correlation between maternal and paternal wage shocks within the household. Such correlation may arise due to assortative mating on unobserved wage determinants or because both parents operate in the same local labor market.

Non-labor income is modeled as a function of both parents' ages and educational attainment levels. In

particular, I first specify the probability that non-labor income is strictly positive in period t as

$$\Pr(I_t > 0) = \frac{\exp(\zeta_t)}{1 + \exp(\zeta_t)}, \tag{35}$$

where

$$\zeta_t = \mu_1 + \mu_2 \cdot age_{1,t} + \mu_3 \cdot age_{1,t}^2 + \mu_4 \cdot ed_1 + \mu_5 \cdot age_{2,t} + \mu_6 \cdot age_{2,t}^2 + \mu_7 \cdot ed_2. \tag{36}$$

Thus, the first part of the non-labor income process is a logit specification with a flexible dependence on both spouses' ages and schooling.

Conditional on non-labor income being strictly positive, its level is specified as

$$\ln I_t = \mu_8 + \mu_9 \cdot age_{1,t} + \mu_{10} \cdot age_{1,t}^2 + \mu_{11} \cdot ed_1 + \mu_{12} \cdot age_{2,t} + \mu_{13} \cdot age_{2,t}^2 + \mu_{14} \cdot ed_2 + \varepsilon_t^I, \tag{37}$$

where

$$\varepsilon_t^I \sim N(0, \sigma_I^2) \tag{38}$$

is assumed to be i.i.d. across households and time. Hence, positive non-labor income realizations follow a log-normal process whose mean depends on the same set of parental age and schooling variables as the wage processes.

B Identification

Exact econometric specification could be found in Section A of the Appendix.

B.1 Preferences and Corporal Punishment Scheme Parameters

In this section, I discuss identification approach to parameters governing preference distribution and corporal punishment scheme usage. Similarly to other structural models focused on dynamic problem of child's human capital formation, providing exact identification conditions is very hard (Del-Boca and Flinn (2009)). Nevertheless, it is possible to convincingly demonstrate that available data provides enough variation to pin down parameters of the model. For this subsection, I treat technology parameters and income process parameters as given and focus only on preferences and corporal punishment scheme parameters.

To obtain intuition on the identification argument, first for simplicity suppose that preferences are non-random. In that case, we have 10 preference parameters from utility functions (1) and (2), as well as disutility of one unit of punishment, δ, and Poisson process parameter of punishment distribution, b. To pin down these parameters, partition available data on household choices at different periods into three sets: 1) choices when no non-compliant leisure is recorded; 2) choices when positive non-compliant leisure is recorded but no corporal punishment scheme is used; 3) choices when positive non-compliant leisure is recorded and corporal punishment scheme is used.

First, focus on the first set of choices where $l_{cd,t} = 0$. Note that within this corner utility terms depending on $l_{cd,t}$ are inactive. That is, conditional on technology parameters and income, marginal rates of substitution within the corner depend only on the parental preferences for leisure (α_1, α_2), consumption (α_3) and cognitive capital (α_4), as well as on the child's preferences for leisure (λ_1) and cognitive capital (λ_3)[6]. As a result, inside

[6]Note that preference for mental stability of both parents (α_5) and child (λ_4) do not affect marginal rates of substitution since

the $l_{cd,t} = 0$ corner the system of choice rules obtained from FOCs can be inverted to obtain preferences parameters up to a normalization.

To illustrate the argument, consider the child's choice problem. Conditional on parental time investments $(\tau_{1,t}, \tau_{2,t})$, recall that the child's remaining time endowment is

$$\widetilde{T}_{c,t} = T_{c,t} - \tau_{1,t} - \tau_{2,t}.$$

When $l_{cd,t} = 0$, the child allocates $\widetilde{T}_{c,t}$ between regular leisure $l_{c,t}$ and self-investment $\tau_{c,t}$. The direct utility term from non-compliant leisure is inactive because

$$\log(1 + l_{cd,t}) = 0.$$

Hence, the relevant intensive-margin problem can be written as

$$\max_{l_{c,t}, \tau_{c,t}} \lambda_1 \log l_{c,t} + \lambda_3 \widetilde{\gamma}_{3,t} \log \tau_{c,t}$$

subject to

$$l_{c,t} + \tau_{c,t} = \widetilde{T}_{c,t}.$$

The first-order condition equates the marginal utility of regular leisure to the marginal value of child self-investment:

$$\frac{\lambda_1}{l_{c,t}} = \frac{\lambda_3 \widetilde{\gamma}_{3,t}}{\tau_{c,t}}.$$

Combining this condition with the time constraint gives

$$l^0_{c,t} = \frac{\lambda_1}{\lambda_1 + \lambda_3 \widetilde{\gamma}_{3,t}} \widetilde{T}_{c,t},$$

and

$$\tau^0_{c,t} = \frac{\lambda_3 \widetilde{\gamma}_{3,t}}{\lambda_1 + \lambda_3 \widetilde{\gamma}_{3,t}} \widetilde{T}_{c,t}.$$

Thus, conditional on the technology term $\widetilde{\gamma}_{3,t}$, and parental time-investments, the child's self-investment choice inside the corner is a function of preference ratio ratio λ_3/λ_1. Analogously, conditional on parental time investments $(\tau_{1,t}, \tau_{2,t})$, the parental problem can be reduced to a system of conditional closed-form choices. Let

$$\widetilde{Y}_t(\tau_{1,t}, \tau_{2,t}) = w_{1,t}(T - \tau_{1,t}) + w_{2,t}(T - \tau_{2,t}) + I_t$$

denote potential household resources after parental time investments, before choosing parental leisure. Also let $\widetilde{\alpha}_{4e,t}$ denote the continuation-value return to productive child expenditure through the child cognitive-capital technology. Conditional on $(\tau_{1,t}, \tau_{2,t})$, the relevant parental choices solve

$$\max_{l_{1,t}, l_{2,t}, c_t, e_t} \alpha_1 \log l_{1,t} + \alpha_2 \log l_{2,t} + \alpha_3 \log c_t + \widetilde{\alpha}_{4e,t} \log e_t,$$

---

under $l_{cd,t} = 0$ $k_{p,t}$ depends on choices this period only through $1_{s,t}$ in TFP.

subject to

$$T = l_{1,t} + h_{1,t} + \tau_{1,t}, \qquad T = l_{2,t} + h_{2,t} + \tau_{2,t},$$

and

$$c_t + e_t = w_{1,t} h_{1,t} + w_{2,t} h_{2,t} + I_t.$$

The first-order conditions imply the conditional closed-form choices

$$c_t(\tau_{1,t}, \tau_{2,t}) = \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3 + \widetilde{\alpha}_{4e,t}} \widetilde{Y}_t(\tau_{1,t}, \tau_{2,t}),$$

$$e_t(\tau_{1,t}, \tau_{2,t}) = \frac{\widetilde{\alpha}_{4e,t}}{\alpha_1 + \alpha_2 + \alpha_3 + \widetilde{\alpha}_{4e,t}} \widetilde{Y}_t(\tau_{1,t}, \tau_{2,t}),$$

$$l_{1,t}(\tau_{1,t}, \tau_{2,t}) = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_3 + \widetilde{\alpha}_{4e,t}} \frac{\widetilde{Y}_t(\tau_{1,t}, \tau_{2,t})}{w_{1,t}},$$

$$l_{2,t}(\tau_{1,t}, \tau_{2,t}) = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \alpha_3 + \widetilde{\alpha}_{4e,t}} \frac{\widetilde{Y}_t(\tau_{1,t}, \tau_{2,t})}{w_{2,t}},$$

and therefore

$$h_{1,t}(\tau_{1,t}, \tau_{2,t}) = T - \tau_{1,t} - l_{1,t}(\tau_{1,t}, \tau_{2,t}),$$

$$h_{2,t}(\tau_{1,t}, \tau_{2,t}) = T - \tau_{2,t} - l_{2,t}(\tau_{1,t}, \tau_{2,t}).$$

The only remaining continuous choices without closed-form solutions are $(\tau_{1,t}, \tau_{2,t})$. After substituting the child's closed-form response and the conditional closed-form parental choices above, the parental problem reduces to a two-dimensional optimization problem:

$$(\tau_{1,t}^0, \tau_{2,t}^0) = \arg\max_{\tau_1, \tau_2} \widetilde{V}_t^0(\tau_1, \tau_2),$$

where $\widetilde{V}_t^0$ is the parental value inside the $l_{cd,t} = 0$ corner after substituting all other choices as functions of $\tau_{1,t}$ and $\tau_{2,t}$.

Note that the resulting system of choices is identical to the one in DFVW (2025). Hence, DFVW (2025) identification arguments apply and the system is invertible in preference parameters up to a normalization. This gives a link to the moments used in estimation. Let A be some set of periods available in the data. The child self-investment moments observed in the data

$$E[\tau_{c,t} \mid l_{cd,t} = 0,\ t \in A]$$

discipline the conditional closed-form child choice rule. The parental time-investment moments

$$E[\tau_{1,t} \mid l_{cd,t} = 0,\ t \in A], \qquad E[\tau_{2,t} \mid l_{cd,t} = 0,\ t \in A]$$

discipline the implicit solution to the reduced parental problem. The labor supply moments

$$E[h_{1,t} \mid h_{1,t} > 0,\ t \in A], \qquad P(h_{1,t} > 0 \mid t \in A),$$

and their analogues for the father discipline the conditional closed-form labor choices. Finally, the expenditure moments

$$E[e_t \mid Y_t \in B], \qquad E[e_t/Y_t \mid Y_t \in B]$$

discipline the conditional closed-form expenditure choice.

Next, consider the second set of choices, where positive non-compliant leisure is recorded but no corporal punishment scheme is used:

$$l_{cd,t} > 0, \qquad 1_{s,t} = 0.$$

This corner is useful because it introduces the preference parameters attached to non-compliant leisure, while still keeping the punishment margin inactive. Relative to the $l_{cd,t} = 0$ corner, the structure of the problem is largely the same. The household still chooses parental time investments, parental labor supply, consumption, productive child expenditure, and the child's time allocation. However, the child's active time-allocation problem now includes non-compliant leisure $l_{cd,t}$ with respective preference weight λ_2, and parents now internalize the disutility from the child's non-compliant leisure, with respective preference weight α_6.

Therefore, conditional on the preference parameters already recovered from the $l_{cd,t} = 0$ corner, the new conditional moment system in the $(l_{cd,t} > 0,\ 1_{s,t} = 0)$ corner needs to recover just two new unknown preference parameters, λ_2 and α_6. For that purpose, in addition to moments analogous to the $l_{cd,t} = 0$ corner, it features a moment for positive non-compliant leisure,

$$E[l_{cd,t} \mid l_{cd,t} > 0,\ 1_{s,t} = 0,\ t \in A].$$

This moment is directly informative about λ_2, because λ_2 governs the child's marginal utility from allocating time to non-compliant leisure. The analogous moments for parental time investments, labor supply, and expenditures are informative about α_6, because α_6 changes the marginal returns to $(\tau_{1,t}, \tau_{2,t})$ through their effect on reducing non-compliant leisure.

Finally, consider the third set of choices, where positive non-compliant leisure is recorded and the corporal punishment scheme is used:

$$l_{cd,t} > 0, \qquad 1_{s,t} = 1.$$

Relative to the $(l_{cd,t} > 0, 1_{s,t} = 0)$ corner, the structure of the problem again remains the same. The household still solves the same choice problem, but now the punishment margin is active. Hence, the child's and parental utilities now contain additional terms associated with mental stability $k_{p,t}$, with preference weights λ_4 and α_5, respectively. In addition, the child's utility function now depends on the direct expected punishment disutility governed by δ, as well as on the unknown parameter b which determines the intensity of expected punishment in Poisson process (4).

Among these new parameters, α_5 is conceptually the most direct extension of the previous identification argument. To see why, suppose for the moment that λ_4, δ, and b are known. Then α_5 is the only new parameter

entering the parental reduced choice system in this corner. When the punishment margin is active and ($l_{cd,t}$ is positive, mental stability directly depends on ($l_{cd,t}$ level and hence on parental time investments ($\tau_{1,t}, \tau_{2,t}$). Therefore, parental utility function now includes additional term which depends on parental time investments ($\tau_{1,t}, \tau_{2,t}$) and hence alters the corresponding marginal returns. Thus, conditional on (λ_4, δ, b) and on the parameters recovered from the first two corners, variation in parental time investments and labor supply in this corner disciplines α_5.

The remaining difficulty is to separate the child's punishment-related parameters. When the punishment scheme is active, realized punishment satisfies

$$s_t \sim \text{Pois}\big(b \log(1 + l_{cd,t})\big),$$

so that

$$E[s_t \mid l_{cd,t}, 1_{s,t} = 1] = b \log(1 + l_{cd,t}).$$

The child therefore faces the direct expected punishment cost

$$-\delta E[s_t] = -\delta b \log(1 + l_{cd,t}).$$

At the same time, realized punishment affects future mental-stability capital, which the child values with preference weight λ_4. After substituting the punishment rule and the continuation-value effect of mental stability, the child's payoff from non-compliant leisure contains a term of the form

$$\left[\lambda_2 - \lambda_3 \widetilde{\gamma}_{4,t} - b\delta - b\lambda_4 \widetilde{\delta}_{p,t}\right] \log(1 + l_{cd,t}).$$

The above term consists of four components: 1) direct utility from non-compliant leisure, λ_2; 2) the cognitive-capital cost of non-compliant leisure, $-\lambda_3 \widetilde{\gamma}_{4,t}$; 3) the direct expected disutility from realized punishment, $-b\delta$; and 4) the indirect cost of punishment through future mental stability, $-b\lambda_4 \widetilde{\delta}_{p,t}$.

Thus, in the child's first-order condition, λ_4, δ, and b all enter through the coefficient on $l_{cd,t}$. The analogous child choice system alone therefore does not separately recover these parameters. It mainly disciplines the composite deterrence term

$$b\left(\delta + \lambda_4 \widetilde{\delta}_{p,t}\right).$$

In other words, without additional information, the child's non-compliant-leisure choice identifies the strength of punishment deterrence, but not separately the direct disutility of punishment δ, the mental-stability valuation λ_4, and the punishment intensity b.

To address this issue, the estimation first separates the punishment-intensity parameter b using a dedicated realized-punishment moment. Since

$$E[s_t \mid l_{cd,t}, 1_{s,t} = 1] = b \log(1 + l_{cd,t}),$$

the conditional moment

$$E[s_t \mid l_{cd,t} > 0,\ 1_{s,t} = 1]$$

disciplines b after plugging in the model-implied choice rule for $l_{cd,t}$, holding fixed the remaining unknown parameters.

Given b, the remaining child choice moments identify the age-specific composite term

$$\delta + \lambda_4 \widetilde{\delta}_{p,t}.$$

To separate δ from λ_4, I rely on variation across periods. The direct disutility from punishment, δ, is constant over time, whereas the continuation-value effect of mental stability, $\widetilde{\delta}_{p,t}$, varies over the child's age. Therefore, moments for positive non-compliant leisure in the punishment corner are computed separately by age bins:

$$E[l_{cd,t} \mid l_{cd,t} > 0,\ 1_{s,t} = 1,\ t \in A], \qquad A \in \{[6,8], [9,12], [13,16]\}.$$

Given the technology parameters identified outside the preference problem, each age-bin moment disciplines the same structural coefficients δ and λ_4, but with a different value of $\widetilde{\delta}_{p,t}$. Thus, any two age-bin moments with distinct values of $\widetilde{\delta}_{p,t}$ provide two equations of the form

$$q_A = \delta + \lambda_4 \widetilde{\delta}_{p,A},$$

where q_A denotes the composite deterrence term implied by the non-compliant-leisure moment in age bin A. Thus, age variation in

$$E[l_{cd,t} \mid l_{cd,t} > 0,\ 1_{s,t} = 1,\ t \in A],$$

together with the realized-punishment moment for b, separates the direct punishment disutility δ from the child's valuation of mental stability λ_4.

This concludes the informal identification argument for the simplified case of homogeneous preferences. However, the actual model uses randomly heterogeneous preferences, which does not allow to conveniently separate parameters into three bins and pinpoint the exact moments for each.

First, note that now by conditioning on behavioral outcomes (e.g., $l_{cd,t} = 0$) we are conditioning on an event which itself depends on preference parameters. As a result, the previous intensive-margin argument no longer works in the same way: we cannot invert the choice system inside a corner and obtain parameters of the preference distribution. With heterogeneous preferences, a conditional moment is affected not only by the choice rule inside the corner, but also by selection into the corner.

For example, a moment such as

$$E[\tau_{c,t} \mid l_{cd,t} = 0]$$

could be expressed as

$$\frac{E\left[\tau^*_{c,t}(\theta_i, X_t)\mathbf{1}\{l^*_{cd,t}(\theta_i, X_t) = 0\}\right]}{P(l^*_{cd,t}(\theta_i, X_t) = 0)},$$

where θ_i denotes the household-specific vector of preference parameters. In the above, indicator

$$\mathbf{1}\{l^*_{cd,t}(\theta_i, X_t) = 0\}$$

depends on the full set of parameters that determine whether the child chooses positive non-compliant leisure.

Thus, conditional choice moments alone are no longer sufficient. The estimation therefore also uses conditional choice probabilities, such as

$$P(l_{cd,t} > 0 \mid 1_{s,t} = 0,\ t \in A), \qquad P(l_{cd,t} > 0 \mid 1_{s,t} = 1,\ t \in A).$$

These probability moments discipline the mass of households whose preference draws place them in each behavioral region, while the previous conditional choice moments discipline the average choices of households inside those regions.

Overall, we have 46 preference and corporal punishment scheme parameters: 8 means and 36 unique entries of variance-covariance matrix of preference distribution, and 2 parameters determining child's disutility from expected punishment. These parameters are pinned down by 81 moments displayed in tables 11 and 12 below.

As could be seen, in addition to conditional means and choice probabilities, I also match variances and covariances of the choice distribution. These moments are useful because preference variance and co-dependence captured by the covariance matrix affects not only the average level of choices, but also their dispersion and co-movements. Additionally, I use variation in age-dependent technology parameters by conditioning moments on different child age bins. Change in technology parameters shifts the marginal returns to time-allocation options and punishment-related choices, while leaving the underlying preference distribution fixed. For labor-supply choices, I also use variation in parental age, since parental age determines the wage and income processes and therefore shifts the opportunity cost of time.

Later, given the estimated set of parameters, I conduct a sensitivity analysis to determine if all estimated parameters of preference distribution meaningfully change the targeted moments. As a result, it was determined that 8 out of 28 covariance parameters have negligible effect on the moments considered. To avoid relying on weakly identified covariance parameters, I impose zero restrictions on these 8 terms. The procedure is described in more details in the Model Estimation section.

Table 22: Choice Moments: Child Time Allocation, Parental Time Investments, Non-compliant Leisure, and Punishment

Moment	Moment	Moment	Moment
$E[\tau_{c,t} \mid l_{cd,t} > 0, t \in [6,8]]$	$E[\tau_{c,t} \mid l_{cd,t} = 0, t \in [6,8]]$	$E[\tau_{c,t} \mid l_{cd,t} > 0, t \in [9,12]]$	$E[\tau_{c,t} \mid l_{cd,t} = 0, t \in [9,12]]$
$E[\tau_{c,t} \mid l_{cd,t} > 0, t \in [13,16]]$	$E[\tau_{c,t} \mid l_{cd,t} = 0, t \in [13,16]]$	$E[\tau_{1,t} \mid l_{cd,t} > 0, t \in [6,8]]$	$E[\tau_{1,t} \mid l_{cd,t} = 0, t \in [6,8]]$
$E[\tau_{1,t} \mid l_{cd,t} > 0, t \in [9,12]]$	$E[\tau_{1,t} \mid l_{cd,t} = 0, t \in [9,12]]$	$E[\tau_{1,t} \mid l_{cd,t} > 0, t \in [13,16]]$	$E[\tau_{1,t} \mid l_{cd,t} = 0, t \in [13,16]]$
$E[\tau_{2,t} \mid l_{cd,t} > 0, t \in [6,8]]$	$E[\tau_{2,t} \mid l_{cd,t} = 0, t \in [6,8]]$	$E[\tau_{2,t} \mid l_{cd,t} > 0, t \in [9,12]]$	$E[\tau_{2,t} \mid l_{cd,t} = 0, t \in [9,12]]$
$E[\tau_{2,t} \mid l_{cd,t} > 0, t \in [13,16]]$	$E[\tau_{2,t} \mid l_{cd,t} = 0, t \in [13,16]]$	$P(I_{s,t} > 0 \mid t \in [6,8], ed_2 \leq 12)$	$P(I_{s,t} > 0 \mid t \in [6,8], ed_2 > 12)$
$P(I_{s,t} > 0 \mid t \in [9,12], ed_2 \leq 12)$	$P(I_{s,t} > 0 \mid t \in [9,12], ed_2 > 12)$	$P(I_{s,t} > 0 \mid t \in [13,16], ed_2 \leq 12)$	$P(I_{s,t} > 0 \mid t \in [13,16], ed_2 > 12)$
$P(l_{cd,t} > 0 \mid t \in [6,8], I_{s,t} = 0)$	$P(l_{cd,t} > 0 \mid t \in [6,8], I_{s,t} = 1)$	$P(l_{cd,t} > 0 \mid t \in [9,12], I_{s,t} = 0)$	$P(l_{cd,t} > 0 \mid t \in [9,12], I_{s,t} = 1)$
$P(l_{cd,t} > 0 \mid t \in [13,16], I_{s,t} = 0)$	$P(l_{cd,t} > 0 \mid t \in [13,16], I_{s,t} = 1)$	$E[l_{cd,t} \mid t \in [6,8], I_{s,t} = 0]$	$E[l_{cd,t} \mid t \in [6,8], I_{s,t} = 1]$
$E[l_{cd,t} \mid t \in [9,12], I_{s,t} = 0]$	$E[l_{cd,t} \mid t \in [9,12], I_{s,t} = 1]$	$E[l_{cd,t} \mid t \in [13,16], I_{s,t} = 0]$	$E[l_{cd,t} \mid t \in [13,16], I_{s,t} = 1]$
$\sigma[\tau_{c,t} \mid t \in [6,8]]$	$\sigma[l_{cd,t} \mid t \in [6,8]]$	$\sigma[\tau_{c,t} \mid t \in [9,12]]$	$\sigma[l_{cd,t} \mid t \in [9,12]]$
$\sigma[\tau_{c,t} \mid t \in [13,16]]$	$\sigma[l_{cd,t} \mid t \in [13,16]]$	$\sigma[\tau_{1,t} \mid t \in [6,16]]$	$\sigma[\tau_{2,t} \mid t \in [6,16]]$
$E[e_t \mid Y_t \in [1000,2500]]$	$E\left[\frac{e_t}{Y_t} \mid Y_t \in [1000,2500]\right]$	$E[s_t \mid t \leq 16, l_{cd,t} > 0, I_{s,t} = 1]$	$\text{corr}(\tau_1, \tau_c)$
$\text{corr}(\tau_2, \tau_c)$	$\text{corr}(\tau_1, \tau_2)$	$\text{corr}(\tau_1, l_{cd})$	$\text{corr}(\tau_2, l_{cd})$
$\text{corr}(\tau_c, l_{cd})$			

Notes: t denotes the child's age, ed_1 denotes the mother's education, and ed_2 denotes the father's education.

Table 23: Choice Moments: Parental Labor Supply

Moment	Moment	Moment	Moment
$E[h_{1,t} \mid h_{1,t} > 0, t_1 \in [18,32]]$	$P(h_{1,t} > 0 \mid t_1 \in [18,32])$	$E[h_{2,t} \mid h_{2,t} > 0, t_2 \in [18,32]]$	$P(h_{2,t} > 0 \mid t_2 \in [18,32])$
$E[h_{1,t} \mid h_{1,t} > 0, t_1 \in [33,37]]$	$P(h_{1,t} > 0 \mid t_1 \in [33,37])$	$E[h_{2,t} \mid h_{2,t} > 0, t_2 \in [33,37]]$	$P(h_{2,t} > 0 \mid t_2 \in [33,37])$
$E[h_{1,t} \mid h_{1,t} > 0, t_1 \in [38,42]]$	$P(h_{1,t} > 0 \mid t_1 \in [38,42])$	$E[h_{2,t} \mid h_{2,t} > 0, t_2 \in [38,42]]$	$P(h_{2,t} > 0 \mid t_2 \in [38,42])$
$E[h_{1,t} \mid h_{1,t} > 0, t_1 \in [43,65]]$	$P(h_{1,t} > 0 \mid t_1 \in [43,65])$	$E[h_{2,t} \mid h_{2,t} > 0, t_2 \in [43,65]]$	$P(h_{2,t} > 0 \mid t_2 \in [43,65])$
$P(h_{1,t} > 0, h_{2,t} > 0 \mid t_1 \in [18,32], t_2 \in [18,32])$	$P(h_{1,t} > 0, h_{2,t} > 0 \mid t_1 \in [33,37], t_2 \in [33,37])$	$P(h_{1,t} > 0, h_{2,t} > 0 \mid t_1 \in [38,42], t_2 \in [38,42])$	$P(h_{1,t} > 0, h_{2,t} > 0 \mid t_1 \in [43,65], t_2 \in [43,65])$
$\sigma[h_{1,t} \mid h_{1,t} > 0, t_1 \in [18,32]]$	$\sigma[h_{2,t} \mid h_{2,t} > 0, t_2 \in [18,32]]$	$\sigma[h_{1,t} \mid h_{1,t} > 0, t_1 \in [33,37]]$	$\sigma[h_{2,t} \mid h_{2,t} > 0, t_2 \in [33,37]]$
$\sigma[h_{1,t} \mid h_{1,t} > 0, t_1 \in [38,42]]$	$\sigma[h_{2,t} \mid h_{2,t} > 0, t_2 \in [38,42]]$	$\sigma[h_{1,t} \mid h_{1,t} > 0, t_1 \in [43,65]]$	$\sigma[h_{2,t} \mid h_{2,t} > 0, t_2 \in [43,65]]$

Notes: t_1 denotes the mother's age, t_2 denotes the father's age.

B.2 Technology Parameters

Given the preferences, corporal punishment scheme parameters, and income-process parameters, the remaining technology parameters are identified from moments involving the evolution of cognitive and mental-stability outcomes. Intuitively, once preferences, corporal-punishment-scheme parameters, and the income process are fixed, the model pins down the distribution of household choices

$$(h_{1,t}, h_{2,t}, \tau_{1,t}, \tau_{2,t}, \tau_{c,t}, l_{cd,t}, e_t, 1_{s,t})$$

at each state. The technology parameters then determine how these inputs are mapped into cognitive and mental-stability capital.

The key idea is that each technology parameter represents the marginal effect of a particular input or state variable on capital accumulation. Therefore, for each technology parameter, I choose a moment that is directly informative about the corresponding input-output relationship. In the actual estimation, these moments are implemented as bivariate regression coefficients rather than raw covariances. This is a natural normalization of the same identifying variation. For example, for a choice variable x_t affecting cognitive capital,

$$\beta_{x_t}^{LW_t} \mid t \in A = \frac{\text{Cov}(LW_t, x_t \mid t \in A)}{\text{Var}(x_t \mid t \in A)}.$$

Importantly, since the CDS waves have five year gaps in-between them, the estimation generally uses both contemporaneous and five-year-forward regression moments. Thus, contemporaneous regression coefficients discipline the cross-sectional input-output relationships observed within the same survey wave, whereas five-year-forward coefficients discipline how current inputs and state variables map into capital observed in the next survey wave. To simplify notation, let

$$\mathcal{A}_0 = \{[6, 8], [9, 12], [13, 16]\}$$

denote the age bins used for contemporaneous moments, and let

$$\mathcal{A}_+ = \{[6, 8], [9, 12]\}$$

denote the age bins used for five-year-forward moments. The final age bin is excluded from $\mathcal{A}_+$, since outcomes at $t + 5$ are not observed for children who are already in the oldest age bin.

For a generic input $x_{j,t}$ of cognitive capital, the relevant moments are

$$\beta_{x_{j,t}}^{LW_t} \mid t \in A, \qquad A \in \mathcal{A}_0,$$

and

$$\beta_{x_{j,t}}^{LW_{t+5}} \mid t \in A, \qquad A \in \mathcal{A}_+.$$

The contemporaneous coefficient captures the cross-sectional relationship between the input and the current level of cognitive capital. The five-year forward coefficient is informative about the dynamic technology, because choices made at t affect capital accumulation between the current CDS wave and the next observed CDS wave.

In particular, for each cognitive capital input

$$x_{j,t} \in \{\tau_{c,t}, l_{cd,t}, e_t\},$$

the model contains three age-bin-specific technology parameters,

$$\gamma_{j,0,g(t)}, \qquad g(t) \in \{1, 2, 3\}.$$

These parameters are disciplined by the corresponding regression moments

$$\beta_{x_{j,t}}^{LW_t} \mid t \in A, \qquad A \in \mathcal{A}_0,$$

together with

$$\beta_{x_{j,t}}^{LW_{t+5}} \mid t \in A, \qquad A \in \mathcal{A}_+.$$

These moments are directly informative about how variation in each input within an age bin is reflected both in current cognitive outcomes and in cognitive outcomes observed in the next survey wave.

For parental time investments ($\tau_{1,t}$ and $\tau_{2,t}$), the technology is richer because the returns to parental time

are allowed to depend on parental education:

$$\gamma_{1,t} = \exp\left(\gamma_{1,0,g(t)} + \gamma_{1,1,g(t)} ed_1\right),$$

and

$$\gamma_{2,t} = \exp\left(\gamma_{2,0,g(t)} + \gamma_{2,1,g(t)} ed_2\right).$$

Thus, for $\tau_{1,t}$ and $\tau_{2,t}$, each age bin contains both a baseline productivity parameter and an education-gradient parameter. For $i \in \{1, 2\}$, the baseline productivity component

$$\gamma_{i,0,g(t)}$$

is disciplined by

$$\beta^{LW_t}_{\tau_{i,t}} \mid t \in A, \qquad A \in \mathcal{A}_0,$$

and

$$\beta^{LW_{t+5}}_{\tau_{i,t}} \mid t \in A, \qquad A \in \mathcal{A}_+.$$

The education-gradient component

$$\gamma_{i,1,g(t)}$$

is disciplined by

$$\beta^{LW_t}_{ed_i} \mid t \in A, \qquad A \in \mathcal{A}_0.$$

Since only the returns to parental time inputs depend on parental education, these moments are directly informative about whether the cognitive return to parental time investment varies systematically with the education of the parent.

The spillover from mental stability to cognitive capital is identified using the same contemporaneous-and-forward logic. Since mental stability enters the cognitive-capital law of motion, the spillover coefficient is disciplined by

$$\beta^{LW_t}_{MS_t} \mid t \in A, \qquad A \in \mathcal{A}_0,$$

and

$$\beta^{LW_{t+5}}_{MS_t} \mid t \in A, \qquad A \in \mathcal{A}_+.$$

The first set of moments captures the contemporaneous relationship between mental stability and cognitive capital, while the second set is more directly informative about the dynamic contribution of current mental stability to future cognitive capital.

The autoregressive components of the two capital laws are disciplined by five-year panel moments. For cognitive capital, the persistence parameter is disciplined by

$$\beta^{LW_{t+5}}_{LW_t} \mid t \in A, \qquad A \in \mathcal{A}_+.$$

These moments capture how strongly current cognitive capital predicts cognitive capital in the next observed

survey wave, after the model accounts for choices and inputs between t and $t+5$. Analogously, the autoregressive component in the mental-stability law of motion is disciplined by

$$\beta_{MS_t}^{MS_{t+5}} \mid t \in A, \qquad A \in \mathcal{A}_+.$$

For the mental-stability technology, the impact of realized punishment is disciplined by regression moments linking mental-stability outcomes to s_t. The coefficient on realized punishment is disciplined by both contemporaneous and five-year-forward moments:

$$\beta_{s_t}^{MS_t} \mid t \in A, \qquad A \in \mathcal{A}_0,$$

and

$$\beta_{s_t}^{MS_{t+5}} \mid t \in A, \qquad A \in \mathcal{A}_+.$$

These moments are directly informative about how variation in executed punishment translates into variation in current and future mental stability.

Finally, the total factor productivity for both capitals is recovered from the average outcome moments. Overall, as specified in Section A.3 of the Appendix, both TFP terms consist of baseline component which is independent of household choices and corporal punishment scheme component.

In the cognitive-capital equation, the baseline TFP component $R_{h,t}(I_{s,t})$ shifts the average level of cognitive capital holding inputs fixed and depends on four unknown d_j parameters. Therefore, moments of the form

$$E[LW_t \mid t], t \in \{6, ..., 16\}$$

are sufficient to discipline the age profile of cognitive-skill productivity.

The effect of the corporal-punishment scheme on cognitive TFP, $\rho_{h,0,g(t)}$, is disciplined by scheme-specific outcome moments,

$$E[LW_t \mid I_{s,t} = 1,\ t \in A], \qquad E[LW_t \mid I_{s,t} = 0,\ t \in A], \qquad A \in \mathcal{A}_0,$$

and

$$E[LW_{t+5} \mid I_{s,t} = 1,\ t \in A], \qquad E[LW_{t+5} \mid I_{s,t} = 0,\ t \in A], \qquad A \in \mathcal{A}_+.$$

For mental stability, the TFP specification is more flexible. The baseline mental-stability TFP component p_t is estimated separately for each period t. This is because average mental-stability scores exhibit period-specific variation that cannot be fully accounted for by $I_{s,t}$, s_t, and lagged mental stability alone. Therefore, moments of the form

$$E[MS_t \mid t], t \in \{6, ..., 16\}$$

discipline the period-specific TFP terms p_t.

The effect of the corporal-punishment scheme on mental-stability productivity is identified from the

similar scheme-specific outcome moments,

$$E[MS_t \mid I_{s,t} = 1,\ t \in A], \qquad E[MS_t \mid I_{s,t} = 0,\ t \in A], \qquad A \in \mathcal{A}_0,$$

and

$$E[MS_{t+5} \mid I_{s,t} = 1,\ t \in A], \qquad E[MS_{t+5} \mid I_{s,t} = 0,\ t \in A], \qquad A \in \mathcal{A}_+.$$

These moments discipline the age-bin-specific scheme effect $\rho_{p,0,g(t)}$.

Thus, given preferences, corporal punishment scheme parameters, and income-process parameters, the technology parameters are pinned down by normalized input-outcome relationships, intertemporal persistence moments, and mean outcome moments.

B.3 Income Process Parameters

The income process consists of wage offers and non-labor income. I estimate the non-labor income process outside of the main model, using the two-step specification described in the Econometric Specification section. First, I estimate a logit model for the probability that non-labor income is strictly positive. Second, conditional on positive non-labor income, I I estimate a log-normal specification for its level. Thus, the non-labor income process is identified directly from observed non-labor income variation outside of the structural estimation.

The wage process is instead estimated jointly with the rest of the model. The reason is that wage offers are observed only for parents who work. Since labor supply is an endogenous choice in the model, the observed wage distribution is endogenously selected: for parents with $h_{i,t} = 0$, the wage offer is censored and not observed in the data. Therefore, the model must jointly match both the endogenous labor-supply decisions and the wage moments computed among working parents. In the simulations, the wage offer process generates potential wage offers for both parents, household choices determine whether each parent works, and wage moments are computed only for simulated observations with positive labor supply, analogously to the data.

The wage offer process for parent $i \in \{1, 2\}$ is

$$\ln w_{i,t} = \eta_{0,i} + \eta_{1,i} age_{i,t} + \eta_{2,i} ed_i + \varepsilon_{i,t},$$

with jointly normally distributed wage shocks:

$$(\varepsilon_{1,t}, \varepsilon_{2,t}) \sim N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\sigma^2_{w1} & \sigma_{w1,w2}\\ \sigma_{w1,w2} & \sigma^2_{w2}\end{bmatrix}\right).$$

Hence, the wage process contains nine parameters: six parameters governing the conditional means of mothers' and fathers' wage offers, and three parameters governing the variance-covariance matrix of wage shocks.

I use 31 wage moments to discipline these 9 wage-process parameters. There are more wage moments than wage-process parameters because the goal is not only to match a just-identified set of moments, but to discipline the entire observed wage-offer distribution relevant for household choices. These moments can be divided into four groups, as specified in Table 13. First, wage means by parental age discipline the age

profile of wage offers for mothers and fathers. Second, wage standard deviations by parental age discipline the dispersion of wage offers over the life cycle. Third, wage means by education and broad age groups discipline the return to education separately for younger and older parents. Finally, the overall wage standard deviations and the correlation between mothers' and fathers' wages discipline the variances and the within-household covariance of wage shocks.

Table 24: Wage Moments Used to Identify the Wage Process

Moment group	**Number of moments**	**Main parameters disciplined**
Wage means by parental age	8	$\eta_{0,i}, \eta_{1,i}$
Wage standard deviations by parental age	8	σ^2_{wi}, age-related dispersion
Wage means by education and broad age	12	$\eta_{2,i}$, education gradients
Overall wage dispersion	2	$\sigma^2_{w1}, \sigma^2_{w2}$
Mother-father wage correlation	1	$\sigma_{w1,w2}$
Total	31	9 wage-process parameters

Table 25: Wage Process Moments

Moment	**Moment**
$\text{corr}(w_{1,t}, w_{2,t})$	$E[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [18, 32]]$
$E[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [33, 37]]$	$E[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [38, 42]]$
$E[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [43, 65]]$	$\sigma[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [18, 32]]$
$\sigma[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [33, 37]]$	$\sigma[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [38, 42]]$
$\sigma[w_{1,t} \mid w_{1,t} > 0,\ t_1 \in [43, 65]]$	$E[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [18, 32]]$
$E[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [33, 37]]$	$E[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [38, 42]]$
$E[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [43, 65]]$	$\sigma[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [18, 32]]$
$\sigma[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [33, 37]]$	$\sigma[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [38, 42]]$
$\sigma[w_{2,t} \mid w_{2,t} > 0,\ t_2 \in [43, 65]]$	$E[w_{1,t} \mid w_{1,t} > 0,\ ed_1 = 12,\ t_1 < 40]$
$E[w_{1,t} \mid w_{1,t} > 0,\ 13 \leq ed_1 \leq 15,\ t_1 < 40]$	$E[w_{1,t} \mid w_{1,t} > 0,\ ed_1 \geq 16,\ t_1 < 40]$
$E[w_{1,t} \mid w_{1,t} > 0,\ ed_1 = 12,\ t_1 \geq 40]$	$E[w_{1,t} \mid w_{1,t} > 0,\ 13 \leq ed_1 \leq 15,\ t_1 \geq 40]$
$E[w_{1,t} \mid w_{1,t} > 0,\ ed_1 \geq 16,\ t_1 \geq 40]$	$E[w_{2,t} \mid w_{2,t} > 0,\ ed_2 = 12,\ t_2 < 40]$
$E[w_{2,t} \mid w_{2,t} > 0,\ 13 \leq ed_2 \leq 15,\ t_2 < 40]$	$E[w_{2,t} \mid w_{2,t} > 0,\ ed_2 \geq 16,\ t_2 < 40]$
$E[w_{2,t} \mid w_{2,t} > 0,\ ed_2 = 12,\ t_2 \geq 40]$	$E[w_{2,t} \mid w_{2,t} > 0,\ 13 \leq ed_2 \leq 15,\ t_2 \geq 40]$
$E[w_{2,t} \mid w_{2,t} > 0,\ ed_2 \geq 16,\ t_2 \geq 40]$	$\sigma[w_{1,t} \mid w_{1,t} > 0]$
$\sigma[w_{2,t} \mid w_{2,t} > 0]$	

Notes: $w_{1,t}$ and $w_{2,t}$ denote, respectively, mother's and father's hourly wages. The variables t_1 and t_2 denote mother's and father's ages, while ed_1 and ed_2 denote mother's and father's educational attainment in years. Wage moments are computed conditional on observed positive wages because wage offers are censored for parents who do not work.

B.4 Model Estimation

The estimator is implemented using simulation methods. Given a trial vector of primitive parameters and the initial observed characteristics of each household, I simulate household histories over the child-development

period. For each household, the simulation draws preferences, wage offers, non-labor income, and the initial state variables. Conditional on these draws and on the trial parameter vector, I solve the household problem and simulate household choices, including labor supply, time investments, expenditures, corporal punishment scheme choices, realized punishment, child time allocation, non-compliant leisure, cognitive capital, and mental stability.

The simulated data is then used to construct the same set of moments as in the observed data. Full set of moments matched could be found in section D of the Appendix. These moments include parental and child time allocation, non-compliant leisure, corporal punishment scheme use, cognitive and mental-stability outcomes, and the empirical relationships among these objects. The parameter estimates are obtained by minimizing the distance between the simulated and observed moments. The distance is weighted by a diagonal weighting matrix, with each moment scaled by an estimate of its sampling variance. The resulting estimator is thus a standard method of simulated moments estimator.

After obtaining an initial set of estimates, I conduct a sensitivity analysis of the preference covariance structure. The purpose of this exercise is to identify preference covariance parameters that are weakly disciplined by the targeted moments. Starting from the estimated model, I perturb each covariance parameter and evaluate the resulting change in the simulated moments and in the objective function. Parameters whose perturbations generate essentially no change in the objective are treated as weakly identified and are restricted in the next estimation step. I then re-estimate the model under the updated restrictions and repeat the same diagnostic procedure. This iteration continues until the remaining unrestricted covariance parameters all have a non-negligible effect on the simulated moments and on the objective function.

This sensitivity procedure can be interpreted as an approximate identification diagnostic. To see this, let $Q(\theta)$ denote the MSM objective function and let ψ denote a covariance parameter. Around the estimated parameter vector $\hat{\theta}$, a first-order Taylor expansion gives

$$Q(\hat{\theta} + \Delta_\psi) \approx Q(\hat{\theta}) + \frac{\partial Q(\hat{\theta})}{\partial \psi} \Delta_\psi.$$

Equivalently, since the objective is a weighted distance between simulated and observed moments, local sensitivity depends on how the simulated moments change with respect to the covariance parameter. If perturbing a covariance parameter does not change the relevant simulated moments, then the first-order change in the objective is close to zero. Such a parameter is locally weakly identified by the chosen moments. Conversely, covariance parameters that generate sizable changes in the simulated moments are locally disciplined by the data.

The final specification therefore retains only those covariance parameters that have meaningful local implications for the targeted moments. This does not constitute a formal global identification proof, but it provides a practical check that the estimated covariance structure is not driven by parameters that are effectively irrelevant for the simulated moments.

C Model Estimates

This section reports the full set of estimated model parameters. Here I collect the underlying parameter estimates, including preference parameters, latent preference heterogeneity, corporal punishment and non-

compliant leisure parameters, technology parameters, non-labor income process estimates, and wage offer process estimates.

Many primitive parameters enter the model through nonlinear transformations. For this reason, some tables report primitive parameters, while others report transformed values that are easier to interpret economically. Whenever relevant, the table notes specify whether the reported entries are primitive estimates or transformed implied values.

C.1 Preference Parameters and Covariance Structure

Preferences are randomly heterogeneous across households. Each household draws a latent preference vector from the estimated normal distribution, and this latent vector is then mapped into parental and child preference weights. See section 6.1 of the main text for the econometric specification. Since the applied transformation is non-linear, parameters of the latent preference distribution could be hard to interpret. Therefore, instead of the latent values, Tables 26 and 27 report the distribution of utility coefficients observed in the simulated population.

Table 26: Distribution of Transformed Preference Parameters

Parameter	**Mean**	**Std. dev.**
A. Parental preferences		
α_1, mother's leisure	0.262	0.074
α_2, father's leisure	0.322	0.025
α_3, consumption	0.269	0.058
α_4, child cognitive capital	0.221	0.020
α_5, child mental stability	0.030	0.003
α_6, disutility from non-compliant leisure	0.085	0.059
B. Child preferences		
λ_1, regular leisure	0.548	0.398
λ_2, non-compliant leisure	0.375	0.416
λ_3, cognitive capital	0.343	0.733
λ_4, mental stability	0.028	0.053

Notes: The table reports moments of simulated transformed household preference coefficients. These are not moments of the latent normal vector.

C.2 Corporal Punishment and Non-compliant Leisure

Table 28 reports the transformed parameters governing realized punishment and the mapping from latent misbehavior to non-compliant leisure (see sections 6.2 and 5.4 of the main text for the econometric specification).

Table 29 reports the primitive parameters of the utility cost of implementing the corporal punishment scheme (see section 6.2 of the main text for the econometric specification).

C.3 Technology of Cognitive and Mental-Stability Capital Formation

Table 30 reports the estimated technology parameters (see section 6.3 of the main text for the econometric specification). The table reports primitive parameters that govern the age and education dependence of the production technologies. For parental time inputs, the table also reports education slopes.

Table 27: Correlation Matrix of Transformed Preference Parameters

	α_1	α_2	α_3	α_4	α_5	α_6	λ_1	λ_2	λ_3	λ_4
α_1	1.000									
α_2	-0.109	1.000								
α_3	-0.612	-0.148	1.000							
α_4	-0.089	0.758	-0.009	1.000						
α_5	-0.089	0.758	-0.009	0.998	1.000					
α_6	-0.590	-0.208	-0.155	-0.291	-0.292	1.000				
λ_1	0.035	0.432	-0.068	0.443	0.443	-0.199	1.000			
λ_2	-0.120	-0.437	0.045	-0.504	-0.505	0.335	-0.933	1.000		
λ_3	0.262	0.018	0.034	0.161	0.161	-0.382	-0.087	-0.271	1.000	
λ_4	0.163	0.152	0.090	0.317	0.317	-0.385	-0.015	-0.315	0.806	1.000

Notes: The table reports correlations among simulated transformed household preference coefficients. Empty entries correspond to the omitted upper triangle of the symmetric correlation matrix.

Table 28: Punishment and Non-compliant Leisure Parameters

Parameter	**Interpretation**	**Estimate**
δ	Disutility from one unit of realized punishment	7.031
b	Poisson intensity of realized punishment	0.139
B	Slope from ξ to $l_{cd,t}$	25.141

Table 29: Utility Cost of Implementing the Corporal Punishment Scheme

	$t \in [6, 8]$	$t \in [9, 12]$	$t \in [13, 16]$
Age-bin intercept, $v_{0,g(t)}$	-18.456	-5.425	-20.057
Education slope, $v_{1,g(t)}$	0.563	0.234	0.526

Notes: The table reports primitive parameters. The implied utility cost is $v_t = \exp(v_{0,g(t)} + v_{1,g(t)} ed_2)$. Positive education slopes imply that the cost of implementing the corporal punishment scheme increases with father's education within each age bin.

Because many primitive technology parameters enter through exponential or other nonlinear transformations, their economic magnitudes are easier to interpret through the implied transformed productivities. Table 31 reports selected transformed parameters evaluated at $ed_1 = ed_2 = 12$. Since the model uses age bins, the values reported at $t = 6$, $t = 10$, and $t = 16$ correspond to the age-bin-specific productivities for $t \in [6, 8]$, $t \in [9, 12]$, and $t \in [13, 16]$, respectively.

Table 32 reports the baseline TFP profiles for cognitive capital and mental stability.

C.4 Non-labor Income and Wage Offer Process

Tables 33 and 34 report the model estimates of non-labor income and wage offer processes, respectively (see section 6.4 of the main text for the econometric specification).

Table 30: Technology Parameter Estimates

	$t \in [6, 8]$	$t \in [9, 12]$	$t \in [13, 16]$	Education slope
A. Cognitive-capital technology				
Child self-investment, $\gamma_{\tau_c,0,g(t)}$	-0.705	-1.016	-0.724	–
Mother's time, $\gamma_{\tau_1,0,g(t)}$	-2.173	-2.272	-2.402	0.044
Father's time, $\gamma_{\tau_2,0,g(t)}$	-3.362	-3.524	-3.687	0.093
Monetary investment, $\gamma_{e,0,g(t)}$	-1.560	-1.566	-1.383	–
Lagged cognitive capital, $\gamma_{h,0,g(t)}$	0.096	-0.102	0.065	–
Mental-stability spillover, $\gamma_{p,0,g(t)}$	-0.574	-3.420	-7.858	–
Direct CP-scheme effect on cognitive TFP, $\rho_{h,0,g(t)}$	0.242	-1.006	-1.718	–
Non-compliant leisure, $\gamma_{l_{cd},0,g(t)}$	-29.030	-15.996	-18.068	–
B. Mental-stability technology				
Expected punishment, $f_{p,0,g(t)}$	-1.077	-2.024	-1.639	–
Direct CP-scheme effect on mental-stability TFP, $\rho_{p,0,g(t)}$	-0.002	-0.003	-0.200	–
Lagged mental stability, $\delta_{p,0,g(t)}$	-0.100	-0.081	-0.097	–
C. Cognitive TFP logistic parameters				
d_1	0.325			
d_2	-0.032			
d_3	-0.037			
d_4	-0.071			

Notes: The table reports primitive technology parameters. The model uses three child-age bins: $g(t) = 1$ for $t \in [6, 8]$, $g(t) = 2$ for $t \in [9, 12]$, and $g(t) = 3$ for $t \in [13, 16]$. Parental time inputs are allowed to depend on parental education.

Table 31: Implied Technology Productivities

	$t = 6$	$t = 10$	$t = 16$
A. Cognitive-capital technology			
Child self-investment productivity, $\gamma_{\tau_c,t}$	0.049	0.052	0.069
Mother's time productivity, $\gamma_{\tau_1,t}$, $ed_1 = 12$	0.028	0.025	0.022
Father's time productivity, $\gamma_{\tau_2,t}$, $ed_2 = 12$	0.015	0.013	0.011
Monetary-investment productivity, $\gamma_{e,t}$	0.030	0.030	0.036
Lagged cognitive capital, $\gamma_{h,t}$	1.100	0.903	1.067
Mental-stability spillover, $\gamma_{p,t}$	0.080	0.005	0.000
Direct CP-scheme effect on cognitive TFP, $\rho_{h,t}$	-0.182	-0.144	-0.245
Effect of non-compliant leisure, $\gamma_{l_{cd},t}$	-3.5×10^{-14}	-1.6×10^{-8}	-2.0×10^{-9}
B. Mental-stability technology			
Effect of expected punishment on mental stability, $\delta_{s,t}$	-0.341	-0.132	-0.194
Direct CP-scheme effect on mental-stability TFP, $\rho_{p,t}$	-0.002	-0.003	-0.200
Lagged mental stability, $\delta_{k_p,t}$	0.905	0.922	0.908

Notes: The table reports transformed values implied by the primitive estimates in Table 30. Parental time productivities are evaluated at $ed_1 = ed_2 = 12$.

D Moment Fit

This appendix reports the fit of the estimated model to the targeted moments. For each moment, the tables present the empirical value and the corresponding simulated value produced by the estimated model. The moments are grouped into five blocks: moments related to non-compliant leisure, the corporal-punishment

Table 32: Baseline TFP Profiles

Age	6	7	8	9	10	11	12	13	14	15	16
Cognitive TFP, $R_{h,t}(1_{s,t} = 0)$	0.167	0.170	0.173	0.176	0.180	0.183	0.186	0.189	0.192	0.195	0.198
Mental-stability TFP, $R_{p,t}(1_{s,t} = 0)$	-0.047	-0.015	0.008	-0.021	-0.017	0.001	-0.030	-0.029	1.822	-0.050	-0.041

Notes: Cognitive TFP follows the smooth age profile implied by the estimated logistic specification. Mental-stability TFP is allowed to vary freely by period.

Table 33: Non-labor Income Process Estimates

Parameter	Discrete component	Continuous component
Intercept	-6.112	6.453
Mother's age, t_1	-0.066	0.064
Mother's age squared, t_1^2	0.00037	-0.00034
Mother's education, ed_1	0.089	-0.109
Father's age, t_2	0.323	-0.186
Father's age squared, t_2^2	-0.00313	0.00286
Father's education, ed_2	0.00228	-0.02584
Conditional residual s.d., σ_I	–	3.090

Notes: The discrete component reports estimates from the logit model for the probability of positive non-labor income. The continuous component reports estimates from the conditional log-normal specification estimated using households with positive non-labor income. The parameter σ_I denotes the standard deviation of the conditional log-income shock.

Table 34: Wage Offer Process Estimates

	Mother	Father
Intercept	0.253	0.283
Age coefficient	0.015	0.016
Education coefficient	0.136	0.164
Shock variance	0.184	0.169
Shock covariance	0.176	

Notes: The table reports wage offer process parameters by parent. Wage offer process is estimated along with the other model parameters since wage censoring is endogenous.

scheme, and time investment choices; cognitive capital; labor supply and wages; mental stability; and miscellaneous second moments of time investments.

D.1 Non-compliant Leisure, Corporal-Punishment Scheme, and Investment Choices

Table 35: Non-compliant Leisure, Corporal-Punishment Scheme, and Investment Moments

Moment	Data	Simulation	Moment	Data	Simulation
$E[s_t \mid t \leq 16, I_{s,t} = 1]$	0.432	0.460	$E[\xi_t \mid t \in [6,8], I_{s,t} = 0]$	0.238	0.146
$E[\xi_t \mid t \in [9,12], I_{s,t} = 0]$	0.253	0.206	$E[\xi_t \mid t \in [13,16], I_{s,t} = 0]$	0.321	0.383
$E[\xi_t \mid t \in [6,8], I_{s,t} = 1]$	0.583	0.543	$E[\xi_t \mid t \in [9,12], I_{s,t} = 1]$	0.437	0.535
$E[\xi_t \mid t \in [13,16], I_{s,t} = 1]$	0.476	0.506	$P(l_{cd,t} > 0 \mid t \in [6,8], I_{s,t} = 0)$	0.447	0.466
$P(l_{cd,t} > 0 \mid t \in [9,12], I_{s,t} = 0)$	0.426	0.484	$P(l_{cd,t} > 0 \mid t \in [13,16], I_{s,t} = 0)$	0.529	0.540
$P(l_{cd,t} > 0 \mid t \in [6,8], I_{s,t} = 1)$	0.641	0.420	$P(l_{cd,t} > 0 \mid t \in [9,12], I_{s,t} = 1)$	0.619	0.452
$P(l_{cd,t} > 0 \mid t \in [13,16], I_{s,t} = 1)$	0.640	0.521	$E[\tau_{c,t} \mid l_{cd,t} > 0, t \in [6,8]]$	1.442	2.878
$E[\tau_{c,t} \mid l_{cd,t} > 0, t \in [9,12]]$	4.176	3.036	$E[\tau_{c,t} \mid l_{cd,t} > 0, t \in [13,16]]$	4.725	4.688
$E[\tau_{c,t} \mid l_{cd,t} = 0, t \in [6,8]]$	1.676	2.354	$E[\tau_{c,t} \mid l_{cd,t} = 0, t \in [9,12]]$	3.492	2.989
$E[\tau_{c,t} \mid l_{cd,t} = 0, t \in [13,16]]$	5.738	5.739	$E[\tau_{c,t} \mid I_{s,t} = 0]$	4.167	3.767
$E[\tau_{c,t} \mid I_{s,t} = 1]$	2.791	2.859	$E[\tau_{1,t} \mid l_{cd,t} > 0, t \in [6,8]]$	16.661	17.125
$E[\tau_{1,t} \mid l_{cd,t} > 0, t \in [9,12]]$	16.921	17.031	$E[\tau_{1,t} \mid l_{cd,t} > 0, t \in [13,16]]$	12.936	13.565
$E[\tau_{1,t} \mid l_{cd,t} = 0, t \in [6,8]]$	18.668	18.228	$E[\tau_{1,t} \mid l_{cd,t} = 0, t \in [9,12]]$	16.824	16.679
$E[\tau_{1,t} \mid l_{cd,t} = 0, t \in [13,16]]$	13.844	12.972	$E[\tau_{2,t} \mid l_{cd,t} > 0, t \in [6,8]]$	7.752	7.593
$E[\tau_{2,t} \mid l_{cd,t} > 0, t \in [9,12]]$	7.906	7.235	$E[\tau_{2,t} \mid l_{cd,t} > 0, t \in [13,16]]$	5.509	5.732
$E[\tau_{2,t} \mid l_{cd,t} = 0, t \in [6,8]]$	7.726	7.886	$E[\tau_{2,t} \mid l_{cd,t} = 0, t \in [9,12]]$	6.317	6.928
$E[\tau_{2,t} \mid l_{cd,t} = 0, t \in [13,16]]$	5.609	5.415	$P(I_{s,t} > 0 \mid t \in [6,8], s_2 \leq 12)$	0.446	0.344
$P(I_{s,t} > 0 \mid t \in [6,8], s_2 > 12)$	0.333	0.306	$P(I_{s,t} > 0 \mid t \in [9,12], s_2 \leq 12)$	0.299	0.328
$P(I_{s,t} > 0 \mid t \in [9,12], s_2 > 12)$	0.212	0.216	$P(I_{s,t} > 0 \mid t \in [13,16], s_2 \leq 12)$	0.158	0.179
$P(I_{s,t} > 0 \mid t \in [13,16], s_2 > 12)$	0.081	0.150	$E[e_t \mid Y_t \in [1000,2500]]$	250.000	468.559
$E[e_t/Y_t \mid Y_t \in [1000,2500]]$	0.210	0.284			

D.2 Cognitive Capital

Table 36: Cognitive Capital Moment Fit

Moment	Data	Simulation	Moment	Data	Simulation
$E[LW_t \mid t = 6]$	23.565	21.839	$E[LW_t \mid t = 7]$	32.121	29.997
$E[LW_t \mid t = 8]$	38.095	36.643	$E[LW_t \mid t = 9]$	41.887	42.335
$E[LW_t \mid t = 10]$	44.655	43.773	$E[LW_t \mid t = 11]$	46.048	45.147
$E[LW_t \mid t = 12]$	47.114	47.158	$E[LW_t \mid t = 13]$	48.737	48.405
$E[LW_t \mid t = 14]$	49.324	50.742	$E[LW_t \mid t = 15]$	50.244	52.387
$E[LW_t \mid t = 16]$	51.333	52.898	$E[LW_t \mid t \in [13, 16]]$	49.759	50.242
$E[LW_t \mid t \leq 8]$	32.018	32.396	$E[LW_t \mid t \in [9, 12]]$	45.004	44.678
$\sigma(LW_t \mid t \in [13, 16])$	4.158	6.236	$\sigma(LW_t \mid t \leq 8)$	8.927	10.501
$\sigma(LW_t \mid t \in [9, 12])$	5.563	8.518	$E[LW_{t+5} \mid t \leq 16, I_{s,t} = 1]$	47.550	45.370
$E[LW_{t+5} \mid t \leq 16, I_{s,t} = 0]$	49.908	50.459	$E[LW_{t+5} \mid t > 8, I_{s,t} = 1]$	48.745	48.658
$E[LW_{t+5} \mid t > 8, I_{s,t} = 0]$	51.059	52.559	$\beta_{LW_t}^{LW_{t+5}} \mid t \leq 16$	0.442	0.675
$\beta_{LW_t}^{LW_{t+5}} \mid t \in [9, 16]$	0.582	0.510	$\beta_{l_{cd,t}}^{LW_{t+5}} \mid t \leq 16$	-0.404	-0.897
$\beta_{l_{cd,t}}^{LW_{t+5}} \mid t \in [9, 16]$	-0.208	-0.755	$\beta_{MS_t}^{LW_{t+5}} \mid t \leq 16$	0.144	0.422
$\beta_{MS_t}^{LW_{t+5}} \mid t \in [9, 16]$	0.181	0.170	$\text{corr}(LW_{t+5}, \tau_{c,t} \mid t \leq 16)$	0.145	0.149
$\text{corr}(LW_{t+5}, \tau_{c,t} \mid t \in [9, 16])$	0.161	0.203	$\text{corr}(\Delta_{t+5,t} LW, \tau_{1,t} \mid t \leq 16)$	0.045	-0.282
$\text{corr}(\Delta_{t+5,t} LW, \tau_{1,t} \mid t \leq 8)$	0.017	-0.187	$\text{corr}(\Delta_{t+5,t} LW, \tau_{2,t} \mid t \leq 16)$	-0.082	-0.228
$\text{corr}(\Delta_{t+5,t} LW, \tau_{2,t} \mid t \leq 8)$	-0.058	-0.153	$\beta_{l_{cd,t}}^{LW_t} \mid t \leq 16$	-0.474	-0.785
$\beta_{MS_t}^{LW_t} \mid t \leq 16$	0.185	0.572	$\text{corr}(LW_t, \tau_{c,t} \mid t \leq 16)$	0.294	0.152
$\text{corr}(LW_t, \tau_{1,t} \mid t \leq 16)$	-0.130	-0.169	$\text{corr}(LW_t, \tau_{2,t} \mid t \leq 16)$	-0.117	-0.146
$E[LW_t \mid t \leq 16, I_{s,t} = 1]$	37.816	41.140	$E[LW_t \mid t \leq 16, I_{s,t} = 0]$	44.863	45.295
$\beta_{s_2}^{LW_t} \mid t \leq 8$	0.183	0.412	$\beta_{s_2}^{LW_t} \mid t \in [9, 12]$	0.655	0.761
$\beta_{s_2}^{LW_t} \mid t \leq 16$	0.381	0.657	$\beta_{s_1}^{LW_t} \mid t \leq 8$	0.583	0.446
$\beta_{s_1}^{LW_t} \mid t \in [9, 12]$	0.730	0.750	$\beta_{s_1}^{LW_t} \mid t \leq 16$	0.669	0.657
$\beta_{s_1}^{\tau_{1,t}} \mid t \leq 8$	0.137	0.560	$\beta_{s_1}^{\tau_{1,t}} \mid t \in [9, 12]$	-0.405	0.567
$\beta_{s_1}^{\tau_{1,t}} \mid t \in [13, 16]$	0.051	0.380	$\beta_{s_2}^{\tau_{2,t}} \mid t \leq 8$	0.118	0.426
$\beta_{s_2}^{\tau_{2,t}} \mid t \in [9, 12]$	-0.046	0.448	$\beta_{s_2}^{\tau_{2,t}} \mid t \in [13, 16]$	0.307	0.365

D.3 Labor Supply and Wages

Table 37: Labor Supply and Wage Moment Fit

Moment	Data	Simulation	Moment	Data	Simulation
$E[h_{1,t} \mid h_{1,t} > 0, t \in [6,8]]$	31.616	29.167	$E[h_{1,t} \mid h_{1,t} > 0, t \in [9,12]]$	30.414	29.053
$E[h_{1,t} \mid h_{1,t} > 0, t \in [13,16]]$	32.932	30.921	$\sigma(h_{1,t} \mid h_{1,t} > 0, t \in [6,8])$	14.637	16.012
$\sigma(h_{1,t} \mid h_{1,t} > 0, t \in [9,12])$	12.838	15.891	$\sigma(h_{1,t} \mid h_{1,t} > 0, t \in [13,16])$	12.863	16.611
$E[h_{2,t} \mid h_{2,t} > 0, t \in [6,8]]$	43.638	42.108	$E[h_{2,t} \mid h_{2,t} > 0, t \in [9,12]]$	44.519	43.968
$E[h_{2,t} \mid h_{2,t} > 0, t \in [13,16]]$	43.780	45.995	$\sigma(h_{2,t} \mid h_{2,t} > 0, t \in [6,8])$	9.886	10.853
$\sigma(h_{2,t} \mid h_{2,t} > 0, t \in [9,12])$	9.791	10.607	$\sigma(h_{2,t} \mid h_{2,t} > 0, t \in [13,16])$	10.622	10.618
$P(h_{1,t} > 0 \mid t \in [6,8])$	0.797	0.806	$P(h_{1,t} > 0 \mid t \in [9,12])$	0.811	0.811
$P(h_{1,t} > 0 \mid t \in [13,16])$	0.846	0.831	$P(h_{2,t} > 0 \mid t \in [6,8])$	0.973	0.985
$P(h_{2,t} > 0 \mid t \in [9,12])$	0.987	0.988	$P(h_{2,t} > 0 \mid t \in [13,16])$	0.990	0.991
$P(h_{1,t} > 0, h_{2,t} > 0 \mid t \in [6,8])$	0.770	0.796	$P(h_{1,t} > 0, h_{2,t} > 0 \mid t \in [9,12])$	0.800	0.804
$P(h_{1,t} > 0, h_{2,t} > 0 \mid t \in [13,16])$	0.836	0.825	$E[w_{1,t} \mid w_{1,t} > 0, t \in [18,32]]$	13.568	13.061
$E[w_{1,t} \mid w_{1,t} > 0, t \in [33,37]]$	17.077	15.654	$E[w_{1,t} \mid w_{1,t} > 0, t \in [38,42]]$	18.961	18.464
$E[w_{1,t} \mid w_{1,t} > 0, t \in [43,65]]$	22.126	22.830	$\sigma(w_{1,t} \mid w_{1,t} > 0, t \in [18,32])$	5.433	7.038
$\sigma(w_{1,t} \mid w_{1,t} > 0, t \in [33,37])$	11.332	8.316	$\sigma(w_{1,t} \mid w_{1,t} > 0, t \in [38,42])$	11.475	9.871
$\sigma(w_{1,t} \mid w_{1,t} > 0, t \in [43,65])$	13.374	12.039	$E[w_{2,t} \mid w_{2,t} > 0, t \in [18,32]]$	20.367	18.894
$E[w_{2,t} \mid w_{2,t} > 0, t \in [33,37]]$	22.668	22.676	$E[w_{2,t} \mid w_{2,t} > 0, t \in [38,42]]$	27.335	26.452
$E[w_{2,t} \mid w_{2,t} > 0, t \in [43,65]]$	32.424	33.876	$\sigma(w_{2,t} \mid w_{2,t} > 0, t \in [18,32])$	11.327	10.986
$\sigma(w_{2,t} \mid w_{2,t} > 0, t \in [33,37])$	13.667	12.748	$\sigma(w_{2,t} \mid w_{2,t} > 0, t \in [38,42])$	16.392	15.704
$\sigma(w_{2,t} \mid w_{2,t} > 0, t \in [43,65])$	21.208	19.646	$E[w_{1,t} \mid w_{1,t} > 0, s_1 = 12, t < 40]$	14.622	12.868
$E[w_{1,t} \mid w_{1,t} > 0, 13 \leq s_1 \leq 15, t < 40]$	16.601	16.199	$E[w_{1,t} \mid w_{1,t} > 0, s_1 \geq 16, t < 40]$	21.440	22.773
$E[w_{1,t} \mid w_{1,t} > 0, s_1 = 12, t \geq 40]$	16.827	14.885	$E[w_{1,t} \mid w_{1,t} > 0, 13 \leq s_1 \leq 15, t \geq 40]$	19.948	18.832
$E[w_{1,t} \mid w_{1,t} > 0, s_1 \geq 16, t \geq 40]$	25.683	27.068	$E[w_{2,t} \mid w_{2,t} > 0, s_2 = 12, t < 40]$	19.533	18.065
$E[w_{2,t} \mid w_{2,t} > 0, 13 \leq s_2 \leq 15, t < 40]$	23.379	25.005	$E[w_{2,t} \mid w_{2,t} > 0, s_2 \geq 16, t < 40]$	33.764	38.171
$E[w_{2,t} \mid w_{2,t} > 0, s_2 = 12, t \geq 40]$	23.784	20.830	$E[w_{2,t} \mid w_{2,t} > 0, 13 \leq s_2 \leq 15, t \geq 40]$	29.140	29.818
$E[w_{2,t} \mid w_{2,t} > 0, s_2 \geq 16, t \geq 40]$	40.268	44.311	$\sigma(w_{1,t} \mid w_{1,t} > 0)$	12.141	10.670
$\sigma(w_{2,t} \mid w_{2,t} > 0)$	18.632	17.812	$\text{corr}(w_{1,t}, w_{2,t})$	0.308	0.881

D.4 Mental Stability

Table 38: Mental-Stability Moment Fit

Moment	Data	Simulation	Moment	Data	Simulation
$E[MS_t \mid t = 6]$	14.543	14.229	$E[MS_t \mid t = 7]$	13.897	13.940
$E[MS_t \mid t = 8]$	14.730	14.190	$E[MS_t \mid t = 9]$	13.694	13.961
$E[MS_t \mid t = 10]$	13.603	13.831	$E[MS_t \mid t = 11]$	13.413	13.621
$E[MS_t \mid t = 12]$	14.171	13.523	$E[MS_t \mid t = 13]$	13.579	13.413
$E[MS_t \mid t = 14]$	13.618	13.136	$E[MS_t \mid t = 15]$	15.067	15.077
$E[MS_t \mid t = 16]$	13.714	14.486	$E[MS_t \mid t \in [13, 16]]$	13.934	13.716
$E[MS_t \mid t \leq 8]$	14.389	14.114	$E[MS_t \mid t \in [9, 12]]$	13.735	13.725
$\sigma(MS_t \mid t \in [13, 16])$	3.636	3.391	$\sigma(MS_t \mid t \leq 8)$	3.067	3.157
$\sigma(MS_t \mid t \in [9, 12])$	3.475	3.408	$\Delta_{I_s}[MS_{t+5} \mid t \leq 16]$	13.825	13.624
$\Delta_{I_s}[MS_{t+5} \mid t \leq 16]$	13.973	13.959	$\Delta_{I_s}[MS_{t+5} \mid t \in [9, 16]]$	14.082	13.820
$\Delta_{I_s}[MS_{t+5} \mid t \in [9, 16]]$	14.042	14.202	$\beta_{MS_t}^{MS_{t+5}} \mid t \leq 16$	0.542	0.646
$\beta_{MS_t}^{MS_{t+5}} \mid t \in [9, 16]$	0.463	0.612	$\beta_{s_t}^{MS_{t+5}} \mid t \leq 16$	-0.598	-1.083
$\beta_{s_t}^{MS_{t+5}} \mid t \leq 8$	-0.770	-0.266	$\beta_{s_t}^{MS_t} \mid t \leq 16$	-0.845	-0.753
$\Delta_{I_s}[MS_t \mid t \leq 16]$	13.533	13.698	$\Delta_{I_s}[MS_t \mid t \leq 16]$	14.115	13.812

D.5 Miscellaneous Time-Investment Moments

Table 39: Variance and Correlations of Time-Investment Choice Variables

Moment	**Data**	**Simulation**	**Moment**	**Data**	**Simulation**
$\sigma(\tau_1)$	10.69	8.93	$\sigma(\tau_2)$	6.60	3.07
$\sigma(\tau_c)$	5.51	7.78	$\sigma(\xi \cdot \mathbf{1}_{l_{cd}>0})$	0.62	0.63
$\text{corr}(\tau_1, \tau_c)$	-0.18	-0.15	$\text{corr}(\tau_2, \tau_c)$	-0.14	-0.13
$\text{corr}(\tau_1, \xi \cdot \mathbf{1}_{l_{cd}>0})$	-0.02	-0.04	$\text{corr}(\tau_2, \xi \cdot \mathbf{1}_{l_{cd}>0})$	0.02	-0.03
$\text{corr}(\tau_c, \xi \cdot \mathbf{1}_{l_{cd}>0})$	-0.07	-0.12	$\text{corr}(\tau_1, \tau_2)$	0.21	0.75